Chapter 1
Accelerated Linear Motion

Section 1A:
The Accelerated Linear Motion Formulae – Horizontal Motion

Displacement is the position of a point relative to an origin.
It is a vector quantity. Displacement can be thought of as distance in a given direction. For example, if a point A is 35 m East of an origin, the displacement of point A is 35 m East, which, if East is taken as the positive direction could be written: $s_A = +35$ m. whereas if point B is 40 m West of the origin, its displacement is: $s_B = -40$ m.

Velocity is the rate of change of displacement with respect to time.
Velocity is a vector quantity. It is usually measured in metres per second (m s^{-1} or m/s). Sometimes velocities are given in kilometres per hour (km h^{-1} or km/hr). Velocity can be thought of as speed in a given direction. If in the example above it had taken 7 seconds to get from the origin to point A, the velocity at point A is $\frac{+35 \text{ m.}}{7 \text{ s.}} = +5 \text{ m s}^{-1}$. whereas the velocity at point B, if it had taken eight seconds to get from the origin to point B, is $\frac{-40 \text{ m.}}{8 \text{ s.}} = -5 \text{ m s}^{-1}$. Note that the sign for a velocity just implies its direction.

Acceleration is the rate of change of velocity with respect to time.
Acceleration is a vector quantity. It is measured in metres per second squared (m s^{-2} or m/s^2). For example, if a body has a velocity of 2 m s^{-1} at point *p*, and has a velocity of 10 m s^{-1} at point *q*, and the time taken to travel from *p* to *q* is 2 s, then the acceleration is $\frac{10 \text{ m s}^{-1} - 2 \text{ m s}^{-1}}{2 \text{ s}} = \frac{8 \text{ m s}^{-1}}{2 \text{ s}} = 4 \text{ m s}^{-2}$ in the direction of the displacement. When a body is slowing down it has a negative acceleration. This is called a **deceleration** or a **retardation**.

Notation. For accelerated linear motion problems, we use the following notation:

- ***u*** **– initial velocity**
- ***v*** **– final velocity**
- ***a*** **– acceleration**
- ***s*** **– displacement**
- ***t*** **– time.**

Note that *u*, *v*, *a* and *s* are all vector quantities, and so can be positive or negative, whereas *t* is a scalar quantity and can therefore only be positive.

Formulae for Accelerated Linear Motion

There are four very useful formulae for accelerated linear motion. You need to know how to use these formulae, and they are the basis of most of the rest of this section of the course.

Formulae for Accelerated Linear Motion:

$$v = u + at$$
$$s = \tfrac{1}{2}(u + v)\,t$$
$$s = ut + \tfrac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$

Notes on using the formulae for Accelerated Linear Motion:

1. For all of these formulae, acceleration is constant, i.e. the acceleration does not change during the motion. When a body is accelerating, a is taken as positive, whereas when it is decelerating, a is taken as negative.
2. For all the vector quantities, u, v, a and s, vectors to the right or upwards are usually taken as positive, whereas vectors to the left or downwards are usually taken as negative.
3. When a body starts from rest, $u = 0$. When a body stops (comes to rest), $v = 0$.
4. At the start of the motion under consideration we usually put $t = 0$ and $s = 0$.

Solution Strategy for accelerated linear motion problems in the horizontal direction:

1) **List out the variables** which you know from u, v, a, s, and t.
2) **Use whichever one of the four constant acceleration formulae** is required to find the required quantity. If you know any three of the quantities, the formulae can be used to find one or both of the other two quantities.

Sometimes we must deal with bodies that are not accelerating, but which are moving with constant velocity. For problems which have constant velocity ($a = 0$), the basic equation is:
distance = velocity x time (s = ut)

A further useful equation is: $\text{average velocity} = \dfrac{\text{total distance}}{\text{total time}}$

Example 1A1

A body starts from rest and accelerates uniformly at 4 m s^{-2}.

(i) Find its velocity after 6 s.
(ii) Convert this velocity into km h^{-1}.
(iii) Find the distance it has travelled in this time.
(iv) If the body then stops accelerating and continues at constant speed, find the time taken until it has gone a total distance of 200 m.

Solution:

(i) $u = 0,\ \ a = 4 \text{ m s}^{-2},\ \ t = 6 \text{ s},\ \ v = ???$

$v = u + at$

$\Rightarrow v = 0 + 4(6) = 24 \text{ m s}^{-1}$

(iii) $u = 0,\ \ a = 4 \text{ m s}^{-2},\ \ t = 6 \text{ s},\ \ s = ???$

$s = ut + \tfrac{1}{2}at^2$

$\Rightarrow s = 0(6) + \tfrac{1}{2}(4)(6)^2 = 72 \text{ m}$

(ii) $24 \text{ m s}^{-1} = 24 \times 60 \times 60 \text{ m h}^{-1} = \dfrac{24 \times 60 \times 60}{1000} \text{ km hr}^{-1} = 86.4 \text{ km h}^{-1}$

(iv) Distance remaining at constant speed $= 200 - 72 = 128$ m.

It is travelling at constant speed of 24 m s^{-1}.

$s = ut$: $\Rightarrow 128 = 24t$, $\Rightarrow t = \dfrac{128}{24} = 5.33$ s.

$\Rightarrow$ total time to travel 200 m $= 6 + 5.33 = 11.33$ s.

Example 1A2

A train is travelling at 144 km h^{-1} as it approaches a station.

(i) Convert 144 km h^{-1} into m s^{-1}.
(ii) What must its deceleration be for it to stop at the station, which is 1000 m away?
(iii) How long does it take the train to come to rest?

Solution:

(i) $144 \text{ km h}^{-1} = 144 \times 1000 \text{ m h}^{-1} = \dfrac{144 \times 1000}{60 \times 60} \text{ m s}^{-1} = 40 \text{ m s}^{-1}$

(ii) $u = 40 \text{ m s}^{-1}$, $v = 0$, $s = 1000$ m, $a = ???$

$v^2 = u^2 + 2as$

$\Rightarrow 0^2 = 40^2 + 2a(1000)$

$\Rightarrow 0 = 1600 + 2000a$

$\Rightarrow 2000a = -1600$

$\Rightarrow a = -0.8 \text{ m s}^{-2}$

or the deceleration is 0.8 m s^{-2}

(iii) $u = 40 \text{ m s}^{-1}$, $v = 0$, $s = 1000$ m, $t = ???$

$s = \tfrac{1}{2}(u + v)t$

$\Rightarrow 1000 = \tfrac{1}{2}(40 + 0)t$

$\Rightarrow 20t = 1000$

$\Rightarrow t = 50$ s

Example 1A3

A car starts from rest and travels along a straight level road with uniform acceleration. It passes in succession 3 telegraph poles spaced 96 m apart. The times between the first and second poles and the second and third poles are 8 s and 4 s, respectively. Find:

(i) the acceleration of the car,
(ii) the speed of the car as it passes the first telegraph pole,
(iii) the distance from the car's starting point to the first telegraph pole.

Solution: (i) Let u be the speed of the car at the first telegraph pole.

From the first pole to the second pole: $u = u \text{ m s}^{-1}$, $t = 8$ s, $s = 96$ m, $a = a \text{ m s}^{-2}$

$s = ut + \tfrac{1}{2}at^2$: $\Rightarrow 96 = 8u + \tfrac{1}{2}a(8)^2$

$\Rightarrow 96 = 8u + 32a$ $[\div 8]$, $\Rightarrow 12 = u + 4a$ **A**

From the first pole to the third pole: $u = u \text{ m s}^{-1}$, $t = 12$ s, $s = 192$ m, $a = a \text{ m s}^{-2}$

$s = ut + \tfrac{1}{2}at^2$: $\Rightarrow 192 = 12u + \tfrac{1}{2}a(12)^2$

$\Rightarrow 192 = 12u + 72a$ $[\div 12]$, $\Rightarrow 16 = u + 6a$ **B**

Now use simultaneous equation to solve these two equations:

A$\times -1$: $-12 = -u - 4a$

B: $16 = u + 6a$

$4 = 2a, \qquad \Rightarrow a = 2 \text{ m s}^{-2}$

(ii) In **A**: $12 = u + 4(2), \qquad \Rightarrow u = 12 - 8 = 4 \text{ m s}^{-1}$

(iii) From the start to the first pole: $u = 0 \text{ m s}^{-1}$, $v = 4 \text{ m s}^{-1}$, $a = 2 \text{ m s}^{-2}$, $s = ???$

$v^2 = u^2 + 2as: \qquad \Rightarrow 4^2 = 0^2 + 2(2)s$

$\Rightarrow 16 = 0 + 4s, \qquad \Rightarrow s = 4 \text{ m.}$

Example 1A4

A car starts from rest with uniform acceleration. In the seventh second of its motion it travels 19.5 m.

(i) Find the acceleration of the car.

(ii) Find the speed of the car at the end of the seventh second.

Solution: (i) The seventh second goes from $t = 6$ s to $t = 7$ s.

For the first 6 s: $u = 0$, $t = 6$ s, $s = s_6$ m, $a = a \text{ m s}^{-2}$

$s = ut + \frac{1}{2}at^2: \qquad \Rightarrow s_6 = 0(6) + \frac{1}{2}a(6)^2 = 18a$

For the first 7 s: $u = 0$, $t = 7$ s, $s = s_7$ m, $a = a \text{ m s}^{-2}$

$s = ut + \frac{1}{2}at^2: \qquad \Rightarrow s_7 = 0(7) + \frac{1}{2}a(7)^2 = 24.5a$

From the question: $s_7 - s_6 = 19.5$

$\Rightarrow 24.5a - 18a = 19.5, \qquad \Rightarrow 6.5a = 19.5, \qquad \Rightarrow a = 3 \text{ m s}^{-2}$

(ii) $u = 0$, $t = 7$ s, $a = 3 \text{ m s}^{-2}$, $v = ???$, $\qquad v = u + at$

$\Rightarrow v = 0 + 3(7) = 21 \text{ m s}^{-1}$

Exercise 1A – using the accelerated linear motion formulae for horizontal motion

1) A lorry accelerates uniformly from rest to 54 km h^{-1} in 10 s.
   (i) Convert 54 km h^{-1} to m s^{-1}.
   (ii) Find its acceleration.
   (iii) Find the distance travelled in this time.
   (iv) If at this instant it stops accelerating and travels on with constant speed for the next 30 seconds, find the total distance travelled.

2) A car accelerates at 2 m s^{-2} from a velocity of 6 m s^{-1} for 4 s.
   (i) Find the distance travelled in this time.
   (ii) Find its velocity after 4 s.
   (iii) If this journey took place in an area with a speed limit of 50 km h^{-1}, has the speed limit been exceeded?

3) A body starting at 8 m s^{-1} accelerates at 2 m s^{-2} while travelling a distance of 240m.
(i) Find the time taken for it to travel 240 m.
(ii) Find the speed reached by the body at this time.
(iii) If the body continues to accelerate at the same rate for a further 10 s, find the total distance travelled.

4) A motorbike, which is travelling at 30 m s^{-1}, starts to decelerate at 3 m s^{-2}.
(i) What is the speed of the motorbike after 3 s?
(ii) How long does it take for the motorbike to come to rest?
(iii) What is the distance travelled by the motorbike in coming to rest?

5) The speed of a train is reduced from 54 km h^{-1} to 36 km h^{-1} over a distance of 125 m.
(i) Show that a speed of 1 km h^{-1} is equivalent to $\frac{5}{18}$ m s^{-1}.
(ii) Calculate the deceleration of the train, assuming it is uniform throughout.
(iii) Find the time taken for the deceleration.
(iv) If the train continues to decelerate at the same rate, calculate how much longer it will take to stop, and also how much farther it will travel before coming to rest.

6) Three points *p*, *q* and *r* lie on a straight level road such that $|pq| = |qr| = 125$ m. A car, travelling with uniform acceleration, passes point *p* with a speed of 10 m s^{-1} and passes point *q* with a speed of 15 m s^{-1}.
(i) Find the uniform acceleration of the car.
(ii) Find the time it takes to travel from *p* to *q*.
(iii) Find the speed of the car as it passes point *r*, giving your answer in the form $p\sqrt{q}$, where $p, q \in \mathbb{N}$.
(iv) The car passes a fourth point *s* with speed 25 m s^{-1}. Find the distance $|rs|$.

7) Four points *a*, *b*, *c* and *d* lie on a straight level road. A car, travelling with uniform retardation, passes point *a* with a speed of 40 m s^{-1} and passes point *b* with a speed of 30 m s^{-1} 4 seconds later. The car comes to rest at *d*.
(i) Find the uniform retardation of the car.
(ii) Find the distance from *a* to *b*.
(iii) Find the distance from *b* to *d*, and also the time taken.
(iv) The car passes point *c* after half the time taken to travel from *b* to *d*. Find the distance between the points *b* and *c*.

8) A truck, which is decelerating uniformly, passes a point at a certain speed *u* m s^{-1} and in the next two seconds travels 44 m, and in the following three seconds it travels a further 51 m.
Find: (i) the deceleration of the truck,
(ii) the value of *u*,
(iii) the total distance travelled by the truck before coming to rest.

9) Two points *p* and *q* lie on a straight stretch of level road. Car A passes the point *p* with a speed of 4 m s^{-1} travelling towards *q* and accelerating uniformly at 2 m s^{-2}. As car A passes point *p*, a second car B is passing point *q* with a speed of 2 m s^{-1} travelling towards point *p* and accelerating uniformly at 3 m s^{-2}. The two cars pass each other at a point *r*, between *p* and *q*, after 8 seconds.

(i) Find the speed of each car at the moment when the two cars pass each other.
(ii) Find the distance each car has travelled during this 8 seconds.
(iii) At the instant the cars pass each other at *r*, car A stops accelerating and travels on to *q* with constant velocity. Calculate the total time car A takes to travel from *p* to *q*.
(iv) After the two cars pass each other at *r*, car B continues to *p* with the same acceleration as before. Find its speed in m s^{-1} as it passes point *p*, correct to one place of decimals. If the speed limit on this stretch of road is 120 km h-1, has car B exceeded the speed limit?

10) Two points *p* and *q* lie on a straight stretch of level road. Car A passes the point *p* with a speed of 2 m s^{-1} travelling towards *q* and accelerating uniformly at 2 m s^{-2}. As car A passes point *p*, a second car B is passing point *q* with a speed of 8 m s^{-1} travelling towards point *p* and accelerating uniformly at 0.5 m s^{-2}. The two cars pass each other after 10 seconds.
(i) Find the distance each car has travelled during this 10 seconds.
(ii) If instead of accelerating at 0.5 m s^{-2}, car B changes its acceleration to *f* m s^{-2}, the two cars now pass each other after 8 seconds instead of 10 s. The initial speed of cars A and B is the same as before, as is the acceleration of car A. Find the value of *f*.

11) A car starts from rest with uniform acceleration. In the tenth second of its motion it travels 19 m.
(i) Find the acceleration of the car.
(ii) Find how long it takes the car to travel 225 m from rest.

12) A lorry starts from rest with uniform acceleration. In the eighth second of its motion it travels 5.625 m.
(i) Find the acceleration of the lorry.
(ii) Find the distance travelled by the lorry in 20 s from rest.

Section 1B: The Accelerated Linear Motion Formulae – Vertical Motion

Any body freefalling in close proximity to the Earth's surface is taken to accelerate towards the Earth with the acceleration due to gravity. The acceleration due to gravity, *g*, is taken as 10 m s^{-2} downwards. (It is actually closer to 9.8 m s^{-2}, but for the purposes of Leaving Certificate Applied Mathematics at Ordinary Level it is taken as 10 m s^{-2}). For vertical motion problems we usually take the upwards direction as positive; therefore *u* is positive and $a = -10$ m s^{-2}. In problems where all the motion is downwards, the positive direction can be taken as downwards, and in this case $a = 10$ m s^{-2}.

When a body is launched in a vertical direction under the influence of gravity, it first of all rises, but is decelerating as it rises. It reaches a top point, where its velocity is instantaneously zero, before accelerating back towards the ground again. Note that when it lands, it is back at its starting point again, so its overall displacement is zero at this point.

Solution Strategy for linear accelerated motion problems in the vertical direction:

1) **List out the variables** which you know from *u, v, a, s,* and *t*. Note that you should usually keep the up direction positive, thus *u* will be positive and *a* will be – 10 m s^{-2}.
2) Use the four **constant acceleration formulae** as required.
   For **maximum height** problems, $v=0$ and use $v^2 = u^2 + 2as$.
   For **time of flight** problems, $s=0$ and use $s = ut + ½at^2$.

Example 1B1

A stone is thrown vertically upwards with a velocity of 30 m s^{-1}.

(i) Find the maximum height reached by the stone, and the time taken to reach maximum height.

(ii) Find the total time the stone is in flight before landing on the ground again.

Solution: (i) When it reaches maximum height, the velocity is zero, i.e. $v = 0$.

$u = 30 \text{ m s}^{-1}, \; a = -10 \text{ m s}^{-2}, \; v = 0, \; s = ???, \; t = ???$

$v^2 = u^2 + 2as$ $v = u + at$

$\Rightarrow 0^2 = 30^2 + 2(-10)s$ $\Rightarrow 0 = 30 - 10t$

$\Rightarrow 0 = 900 - 20s$ $\Rightarrow 10t = 30$

$\Rightarrow s = 45 \text{ m.}$ $\Rightarrow t = 3 \text{ s.}$

(ii) When it lands on the ground again, its displacement is zero, i.e. $s = 0$.

$u = 30 \text{ m s}^{-1}, \; a = -10 \text{ m s}^{-2}, \; s = 0, \; t = ???$ $s = ut + \frac{1}{2}at^2$

$\Rightarrow 0 = 30t + \frac{1}{2}(-10)t^2,$ $\Rightarrow 5t^2 - 30t = 0$ $[\div 5]$

$\Rightarrow t^2 - 6t = t(t-6) = 0,$ $\Rightarrow t = 0 \quad \text{or} \quad t = 6 \text{ s.}$

N.B. Note that the time of flight, 6 s, is twice the time taken to reach maximum height. This is always true, when a particle lands in the same place as it was projected from.

Example 1B2

A stone is thrown vertically upwards from the top of a cliff 60 m high with a velocity of 20 m s^{-1}.

(i) How long is the stone in flight before it lands on the ground at the base of the cliff?

(ii) Find the speed with which the stone hits the ground at the base of the cliff?

(iii) Through what distance has the stone travelled while in flight?

Solution: (i) First look at the motion of the stone from the point of projection to its highest point. As there is upward motion, take the positive direction as up.

$u = 20 \text{ m s}^{-1}, \; a = -10 \text{ m s}^{-2}, \; v = 0, \; t = ???, \; s = ???$

$v = u + at$

$\Rightarrow 0 = 20 - 10t,$ $\Rightarrow t = 2 \text{ s.}$

$s = ut + \frac{1}{2}at^2$

$\Rightarrow s = 20(2) + \frac{1}{2}(-10)(2)^2 = 40 - 20 = 20 \text{ m.}$

Next look at the motion of the stone from its highest point till it lands at the base of the cliff. As the motion is now only downwards, take the positive direction as down. The stone now has to fall a total distance of $20+60=80$ m.

$u=0 \text{ m s}^{-1}$, $a=10 \text{ m s}^{-2}$, $s=80$ m, $t=???$

$s=ut+\frac{1}{2}at^2$

$\Rightarrow 80=0(t)+\frac{1}{2}(10)t^2$

$\Rightarrow 80=5t^2$

$\Rightarrow t=\sqrt{16}=4$ s.

Therefore the total time from the point of projection to the base of the cliff is $2+4=6$ s.

(ii) To find the speed at the bottom of the cliff, consider the last section of the motion, from the highest point to the bottom of the cliff.

$u=0 \text{ m s}^{-1}$, $a=10 \text{ m s}^{-2}$, $t=4$ s, $v=???$

$v=u+at$

$\Rightarrow v=0+10(4)=40 \text{ m s}^{-1}$.

(iii) We have already calculated that the distance from the point of projection to the highest point is 20 m, and it is 80 m from there to the base of the cliff, so the total distance travelled is $20+80=100$ m.

Exercise 1B – using the accelerated linear motion formulae for vertical motion

1) A ball is thrown vertically upwards with a speed of 36 m s^{-1}.
(i) Find the greatest height reached by the ball, and the time taken to reach the greatest height.
(ii) Find the total time for which the ball is in flight.
(iii) Find the speed of the ball when it lands.

2) A ball is thrown up vertically and reaches a maximum height of 45 m. Find the initial velocity of the ball.

3) A stone is thrown vertically upwards on level ground, and lands again in the same spot after 5 s. What was the initial velocity of the stone?

4) A stone is thrown up with an initial velocity of 40 m s^{-1}.
(i) How long does it take the stone to reach its maximum height, and what is this height?
(ii) What is its velocity and displacement after 5 s?
(iii) What distance has the stone travelled in this time?

5) A stone is dropped from the top of a cliff 20 m high into the sea. Find the speed of the stone as it hits the sea, and the time taken for it to hit the sea.

6) A ball is thrown downwards with a velocity of 10 m s^{-1} from the top of a building 26.25 m high. Find the speed with which the ball hits the ground at the bottom of the building, and find also the time it takes the ball to hit the ground.

7) A stone is thrown vertically upwards from the top of a building 15 m high with a velocity of 10 m s^{-1}.
(i) Find the greatest height reached by the stone above its point of projection, and the time this takes.
(ii) Find the total time it takes the stone to reach the ground from when it was projected upwards.
(iii) Find the speed with which the stone hits the ground.
(iv) Find the total distance travelled by the stone.

8) A stone is thrown up from the top of a cliff with a velocity of 25 m s^{-1}. It lands in the water at the bottom of the cliff 7 s later. Find the height of the cliff.

9) A particle is dropped from rest at a height *h* m above the ground. In the last 0.2 s before it hits the ground, it falls a distance of 4.6 m. Find *h*.

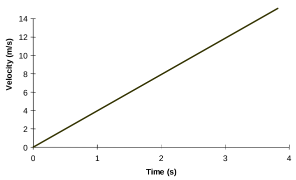
Section 1C: Standard Velocity / Time Graphs

Velocity / time graphs show how the velocity of a moving object varies with respect to time. They are a very useful tool in solving many kinds of motion problems. They are usually used when the motion of a body involves two or more of the following sections:

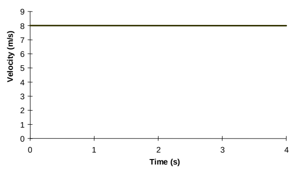
1) a section with constant acceleration
2) a section with constant velocity (no acceleration)
3) a section with constant deceleration

Common Velocity / Time Graph Types

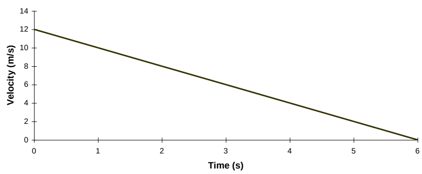
Constant Acceleration



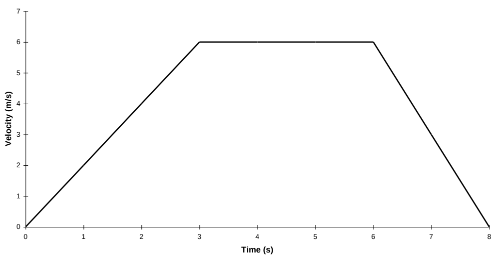
Constant Velocity



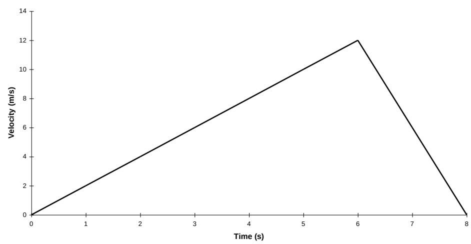
Constant Deceleration



Rest to rest with constant velocity



Rest to rest without constant velocity



Useful formulae for velocity / time graph problems:
On a velocity / time graph,
1) the **slope** of the graph corresponds to the **acceleration** (usually easier to use $v = u + at$)
2) the **area** under the graph corresponds to the **distance travelled**

This second fact, that the area under a velocity / time (VT) graph corresponds to the distance travelled is used in all questions of this kind. As can be seen from the sample VT graphs above, a number of different shapes occur frequently, for which the area must be calculated.

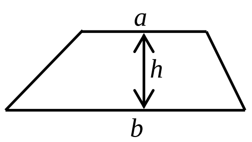
- For a **rectangle**: area = base x height
- For a **triangle**: area = ½ x base x height
- For a **trapezium** (such as the rest to rest with constant velocity graph above):

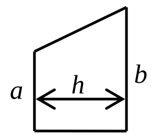
$$\text{area} = \frac{(\text{sum of the lengths of the two parallel sides})}{2} \times \text{ perpendicular distance between the parallel sides}$$

N.B. A **trapezium** is any quadrilateral (four sided shape) with one pair of parallel sides

Area of a trapezium:

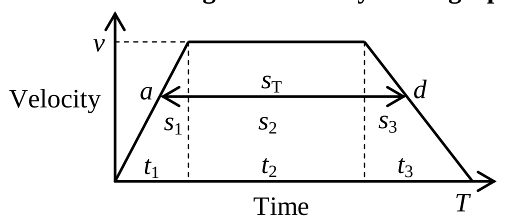
Area of trapezium = $\left(\frac{a+b}{2}\right)h$





This formula for the area of a trapezium is given on page 8 of the book of formulae and tables.

Standard labelling for a velocity / time graph



a – acceleration
d – deceleration
v – constant velocity
T – total time
s_T – total distance
t_1, t_2, t_3 – section times
s_1, s_2, s_3 – section distances

Solution Strategy for velocity / time graph questions:
1) **Sketch the graph** clearly, including all information given in the question.
2) Use $\boldsymbol{v = u + at}$ (or the slope) to involve any **accelerations** and **decelerations**. With graph questions it is better not to use the other constant acceleration formulae.
3) Use the **area under the graph** to bring in the **distance** travelled.
4) It is often useful to use the formula: $\boldsymbol{T = t_1 + t_2 \,(+ t_3)}$

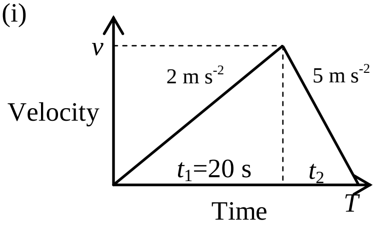
Example 1C1

A train leaves from rest at one station and accelerates at 2 m s^{-2} for 20 seconds until it reaches a certain speed v m s^{-1}. It then immediately starts to decelerate at 5 m s^{-2} and comes to rest at the next station.

(i) Sketch a velocity/time graph for the motion of the train.
(ii) Find v.
(iii) Find the time the train took to decelerate back to rest at the second station.
(iv) Find the distance between the two stations.

Solution:

(i)



(ii) For the acceleration:

$v = u + at$

$\Rightarrow v = 0 + 2(20) = 40 \text{ m s}^{-1}$

(iii) For the deceleration:

$v = u + at$

$\Rightarrow 0 = 40 - 5t_2, \qquad \Rightarrow t_2 = 8 \text{ s}.$

(iv) From the area of the large triangle:

$s = \frac{1}{2}Tv \qquad (\text{Area} = \frac{1}{2} \times \text{base} \times \text{height})$

$\Rightarrow s = \frac{1}{2}(20 + 8)(40) = 560 \text{ m}.$

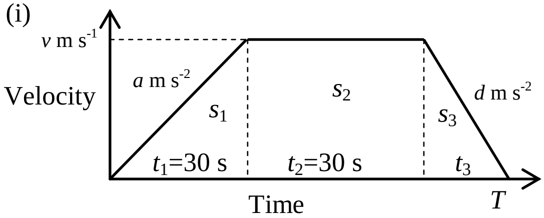
Example 1C2

A car starts from rest on a straight level road. It accelerates uniformly for 30 s, covering 675 m in that time, to reach a speed v m s^{-1}. It then travels at constant speed v m s^{-1} for a further 30 s, before decelerating uniformly back to rest again in a distance of 337.5 m.

(i) Sketch a velocity/time graph for the motion of the train.
(ii) Find v.
(iii) Find the acceleration and deceleration of the car.
(iv) Find the distance travelled at constant speed.
(v) Find the average speed of the car over the whole journey.

Solution:

(i)



For the deceleration, from the area of the triangle: $s_3 = \frac{1}{2}t_3 v$,

$\Rightarrow 337.5 = \frac{1}{2}t_3(45), \quad \Rightarrow t_3 = \frac{337.5}{22.5} = 15 \text{ s}$

$v = u + at, \qquad \Rightarrow 0 = 45 - 15d,$

$\Rightarrow d = 3 \text{ m s}^{-2}$

(ii) For the acceleration, from the area of the triangle:

$s_1 = \frac{1}{2}t_i v, \qquad \Rightarrow 675 = \frac{1}{2}(30)v, \qquad \Rightarrow v = \frac{675}{15} = 45 \text{ m s}^{-1}$

(iii) For the acceleration: $u = 0$, $v = 45 \text{ m s}^{-1}$, $t = 30$ s, $a = ???$

$v = u + at, \qquad \Rightarrow 45 = 0 + 30a, \qquad \Rightarrow a = 1.5 \text{ m s}^{-2}$

(iv) From the area of the rectangle:

$s_2 = t_2 v = 30(45) = 1350$ m.

(v) From the area of the trapezium:

$$\text{Total distance} = \left(\frac{t_2 + T}{2}\right)v = \left(\frac{30 + (30 + 30 + 15)}{2}\right)45 = 2362.5 \text{ m}$$

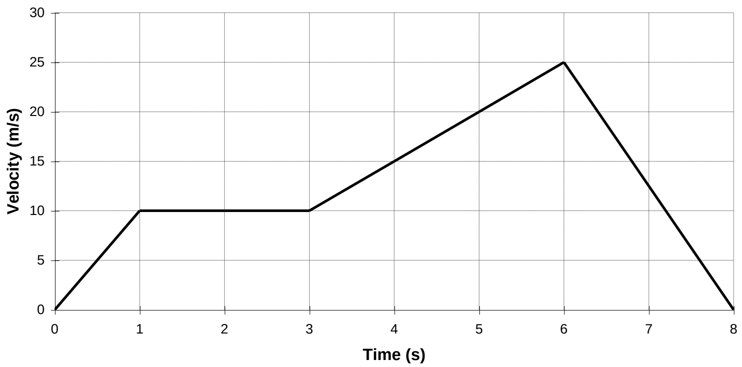
$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{2362.5}{75} = 31.5 \text{ m s}^{-1}$$

Exercise 1C – standard velocity / time graphs

1) A lift descends from rest with constant acceleration of 1 m s^{-2} for the first part of its journey, until it has travelled 18 m, and then decelerates back to rest with a uniform deceleration of 0.75 m s^{-2}.
   (i) Sketch a velocity/time graph for the motion of the lift.
   (ii) Find the time taken for the acceleration.
   (iii) Find the maximum speed reached by the lift.
   (iv) Find the time taken for the deceleration.
   (v) Find the total distance travelled by the lift.

2) A racing car accelerates at 8 m s^{-2} from rest for 7 seconds, and then immediately puts on the brakes and decelerates uniformly back to rest in 4 seconds.
   (i) Sketch a velocity/time graph for the motion of the racing car.
   (ii) Find the maximum speed reached by the car.
   (iii) Find the deceleration of the car while braking.
   (iv) Find the total distance travelled by the racing car.

3) The graph below shows the velocities of a particle at the end of successive seconds. By dividing the graph up into suitable trapezia, rectangles and triangles, find the total distance travelled by the particle, and its average speed over the whole journey.



4) A bus accelerates uniformly from rest at 2 m s^{-2} for 10 s, to reach a speed v m s^{-1}. It travels at this constant speed v for 26 s, before decelerating back to rest in 4 s.
(i) Sketch a velocity / time graph of this motion.
(ii) Find v.
(iii) Find the total distance travelled.
(iv) Find the average speed of the bus.

5) A lorry accelerates uniformly along a straight level road from rest at 0.5 m s^{-2}, until it has travelled 256 m. It then travels at the constant speed reached for 800 m, before braking and decelerating uniformly to rest at 2 m s^{-2}.
(i) Sketch a velocity / time graph of the motion of the lorry.
(ii) Find the maximum speed reached by the lorry.
(iii) Find the total time for the journey.
(iv) Find the total distance travelled in kilometres.

6) On a certain straight level stretch of road there is a speed limit of 25 m s^{-1}. A car accelerates from rest uniformly along this road for 31.25 s, until it reaches the speed limit, and then travels at a speed equal to the speed limit for a distance which is four times that covered while accelerating. It then decelerates to rest, covering a distance of 62.5 m while decelerating.
(i) Sketch a velocity / time graph of the motion of the car.
(ii) Find the acceleration and the deceleration of the car.
(iii) Find the total time for the journey.
(iv) Find the total distance travelled, to the nearest metre.

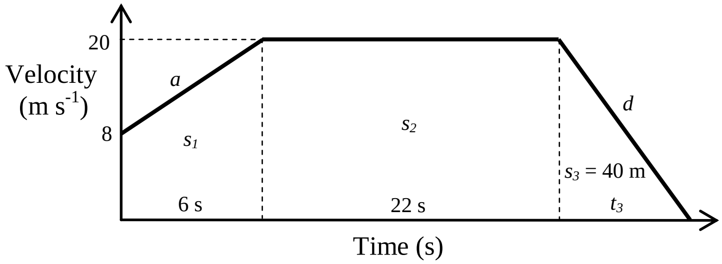
7) A train starts from rest at station A with uniform acceleration of 0.2 m s^{-2}. After it has attained full speed it moves uniformly for 3 minutes. It is brought to rest at station B by the brakes, which apply a constant retardation of 1.5 m s^{-2} for 20 s.
(i) Sketch a velocity / time graph of the motion of the train.
(ii) Find the maximum speed reached by the train.
(iii) Find the total time for the journey.
(iv) Find $|AB|$, in kilometres.
(v) Find the speed of the train when it is 1 km from station A.

Section 1D: Non-Standard Velocity / Time Graphs

In the previous section, velocity / time graphs which had the standard shapes, i.e. triangles and trapezia, were dealt with. Velocity / time graphs can be used to describe a wider variety of motions, as long as they consist of any number of successive sections, each of which is either uniform acceleration, constant velocity or uniform deceleration (retardation). Any motion which consists of two or more such sections is best dealt with by sketching a velocity / time graph of the motion, and then using the area under the graph to work out any distances, and using $v = u + at$ (or the slope of the graph) to deal with any accelerations or decelerations. The example below, and the following exercise, demonstrate typical questions of this sort.

Example 1D1
A car travels along a straight level road. It passes a point A at a speed of 8 m s^{-1} and accelerates uniformly for 6 seconds to a speed of 20 m s^{-1}. It then travels at a constant speed of 20 m s^{-1} for 22 seconds. Finally the car decelerates uniformly from 20 m s^{-1} to rest at a point B. The car travels 40 m while decelerating. Sketch a velocity / time graph for the motion, and hence find:
(i) the acceleration,
(ii) the deceleration,
(iii) $|AB|$, the distance from A to B,
(iv) the average speed of the car as it travels from A to B.

Solution:



(i) For the acceleration:

$u = 8 \text{ m s}^{-1}$, $v = 20 \text{ m s}^{-1}$, $t = 6$ s, $a = ???$

$v = u + at$

$\Rightarrow 20 = 8 + 6a, \qquad \Rightarrow a = \dfrac{20-8}{6} = 2 \text{ m s}^{-2}$

(ii) For the deceleration, from the area of the triangle:

$s = \frac{1}{2}tv: \qquad \Rightarrow 40 = \frac{1}{2}t_3(20), \qquad \Rightarrow t_3 = 4 \text{ s}$

$v = u + at: \qquad \Rightarrow 0 = 20 - 4d, \qquad \Rightarrow d = 5 \text{ m s}^{-2}$

(iii) Need to find s_1 and s_2.

For s_1, from the area of the trapezium: $\Rightarrow s_1 = \left(\dfrac{8+20}{2}\right)6 = 84 \text{ m}$

For s_2, from the area of the rectangle: $\Rightarrow s_2 = 22 \times 20 = 440 \text{ m}$

$\Rightarrow$ total distance $= |AB| = 84 + 440 + 40 = 564$ m.

(iv) Total time $= 6 + 22 + 4 = 32$ s.

Average speed $= \dfrac{\text{Total distance}}{\text{Total time}} = \dfrac{564}{32} = 17.625 \text{ m s}^{-1}$.

Exercise 1D – non-standard velocity / time graphs

1) A motorbike travels along a straight level road. It passes a point P with speed 6 m s^{-1} and accelerates uniformly for 6 seconds to a speed of 24 m s^{-1}. It then moves at a constant speed of 24 m s^{-1} for 25 seconds. Finally the motorbike decelerates uniformly from 24 m s^{-1} to rest at point Q in 4 seconds. Sketch a velocity / time graph for the motion, and hence find:
   (i) the acceleration,
   (ii) the deceleration,
   (iii) $|PQ|$, the distance from P to Q,
   (iv) the average speed of the motorbike for the journey.

2) A truck travels along a straight level road. It passes a point P with speed 5 m s^{-1} and accelerates uniformly for 10 seconds to a speed of 20 m s^{-1}. It then moves at a constant speed of 20 m s^{-1} for 12 seconds. Finally the truck decelerates uniformly from 20 m s^{-1} to rest at point Q. The truck travels 80 m while decelerating. Sketch a velocity / time graph for the motion, and hence find:
   (i) the acceleration,
   (ii) the deceleration,
   (iii) $|PQ|$, the distance from P to Q,
   (iv) the speed of the truck when it is 45 m from Q.

3) A car travels along a straight level road. It passes a point P with speed 10 m s^{-1} and accelerates uniformly for 4 seconds to a speed of 16 m s^{-1}. It then decelerates uniformly to a speed of 7 m s^{-1} and travels 34.5 m during this deceleration. Finally the car maintains a constant speed of 7 m s^{-1} for 8 seconds and then passes Q. Sketch a velocity / time graph for the motion, and hence find:
   (i) the acceleration,
   (ii) the deceleration,
   (iii) $|PQ|$, the distance from P to Q,
   (iv) the average speed of the car for the journey.

4) A train is travelling at a speed of 30 m s^{-1} on a straight level track. As it passes a point P, the driver sees a red signal at S, a distance of 403.5 m ahead of him. He immediately applies the brakes, and the train decelerates at 2 m s^{-2}. At Q, which is 214.5 m from S, the train stops decelerating and it travels at constant speed for 10 seconds until it reaches point R. At that time, as the train passes R, the signal changes to green, and the train immediately accelerates, and it just reaches 30 m s^{-1} as it passes point S. Sketch a speed / time graph for the motion of the train from P to S, and hence:
   (i) find the speed of the train between Q and R,
   (ii) find the distance from R to S, and hence find the acceleration of the train.

5) Two points P and Q lie on a straight level road. A car passes point P with a constant speed of 20 m s^{-1} and continues at this speed for 12 seconds. The car then accelerates uniformly for 5 seconds to a speed of 30 m s^{-1}. Finally the car decelerates uniformly from 30 m s^{-1} to rest at point Q. The car travels 75 m while decelerating. Sketch a velocity / time graph for the motion, and hence find:
   (i) the acceleration,
   (ii) the deceleration,
   (iii) $|PQ|$, the distance from P to Q,
   (iv) the average speed of the car as it travels from P to Q.

6) The points *P* and *Q* lie on a straight level road. A motorbike passes point *P* with a speed of 30 m s^{-1} and decelerates uniformly for 3 seconds to a speed of 9 m s^{-1}. The motorbike now accelerates uniformly from 9 m s^{-1} to a speed of 33 m s^{-1}. The motorbike travels 252 m while accelerating. It then continues at a constant speed of 33 m s^{-1} for 10 seconds and then passes *Q*. Sketch a velocity / time graph for the motion, and hence find:
   (i) the deceleration,
   (ii) the acceleration,
   (iii) |*PQ*|, the distance from *P* to *Q*,
   (iv) the average speed of the motorbike for the journey.

   If there is a legal speed limit of 120 km h^{-1} on this stretch of road, investigate if the motorbike has exceeded the speed limit in the above motion from *P* to *Q*.

7) A car travels along a straight level road. It passes a point *P* with speed 8 m s^{-1} and accelerates uniformly for 4 seconds to a speed of 20 m s^{-1}. It then moves at a constant speed of 20 m s^{-1} for 11 seconds. Finally the car decelerates uniformly from 20 m s^{-1} to rest at point *Q* in 5 seconds. Sketch a velocity / time graph for the motion, and hence find:
   (i) the acceleration,
   (ii) the deceleration,
   (iii) |*PQ*|, the distance from *P* to *Q*.

   A second car travels the same distance from *P* to *Q* in the same overall time as the first car. This car starts from rest at *P* and accelerates uniformly to a maximum speed v m s^{-1}, and then immediately decelerates uniformly to rest at *Q*.
   (iv) Find *v*, the maximum speed reached by the second car.

8) A car travels along a straight level road. It passes a point *P* with speed 8 m s^{-1} and accelerates uniformly for 8 seconds to a speed of 24 m s^{-1}. It then moves at a constant speed of 24 m s^{-1} for 10 seconds. Finally the car decelerates uniformly from 24 m s^{-1} to rest at point *Q* in 6 seconds. Sketch a velocity / time graph for the motion, and hence find:
   (i) the acceleration,
   (ii) the deceleration,
   (iii) |*PQ*|, the distance from *P* to *Q*.

   A second car, with acceleration and deceleration the same as in (i) and (ii) above, starts from rest at *P* and accelerates uniformly to its maximum speed of 20 m s^{-1}. It continues with this maximum speed for a certain time and then decelerates uniformly to rest at *Q*.
   (iv) How long does it take this second car to go from *P* to *Q*.

9) (a) Car A, accelerating uniformly, passes a point *P* with a speed of 8 m s^{-1}. 12 seconds later it reaches a point *Q* with a speed of 20 m s^{-1}. Sketch a speed / time graph for the motion of car A between *P* and *Q*, and hence find the acceleration of car A, and find also |*PQ*|, the distance from *P* to *Q*.
   (b) At the same time as A passes *P*, a second car B also passes point *P*, but with an initial speed of 11 m s^{-1}, and it accelerates uniformly to arrive at *Q* at exactly the same time as A. Sketch a speed / time graph for the motion of car B between *P* and *Q*, and find the speed of car B as it passes point *Q*, and hence find the acceleration of car B.

Answers to Exercises:

Exercise 1A

1) (i) 15 m s^{-1}, (ii) 1.5 m s^{-2}, (iii) 75 m, (iv) 525 m. 2) (i) 40 m, (ii) 14 m s^{-1}, (iii) Yes.
3) (i) 12 s, (ii) 32 m s^{-1}, (iii) 660 m. 4) (i) 21 m s^{-1}, (ii) 10 s, (iii) 150 m. 5) (ii) 0.5 m s^{-2},
5) (iii) 10 s, (iv) 20 s, 100 m. 6) (i) 0.5 m s^{-2}, (ii) 10 s, (iii) $5\sqrt{14}$ m s^{-1}, (iv) 275 m.
7) (i) -2.5 m s^{-2}, (ii) 140 m, (iii) 180 m, 12 s, (iv) 135 m. 8) (i) -2 m s^{-2}, (ii) 24, (iii) 144 m.
9) (i) 20 m s^{-1}, 26 m s^{-1}, (ii) 96 m, 112 m, (iii) 13.6 s, (iv) 35.4 m s^{-1}, Yes.
10) (i) 120 m, 105 m, (ii) 2.53 m s^{-2}. 11) (i) 2 m s^{-2}, (ii) 15 s. 12) (i) 0.75 m s^{-2}, (ii) 150 m.

Exercise 1B

1) (i) 64.8 m, 3.6 s, (ii) 7.2 s, (iii) 36 m s^{-1}. 2) 30 m s^{-1}. 3) 25 m s^{-1}. 4) (i) 4 s, 80 m,
4) (ii) 10 m s^{-1} downwards, 75 m, (iii) 85 m. 5) 20 m s^{-1}, 2 s. 6) 25 m s^{-1}, 1.5 s.
7) (i) 5 m, 1 s, (ii) 3 s., (iii) 20 m s^{-1}, (iv) 25 m. 8) 70 m. 9) 28.8 m.

Exercise 1C

1) (ii) 6 s, (iii) 6 m s^{-1}, (iv) 8 s, (v) 42 m. 2) (ii) 56 m s^{-1}, (iii) 14 m s^{-2}, (iv) 308 m.
3) 102.5 m, 12.8125 m s^{-1}. 4) (ii) 20 m s^{-1}, (iii) 660 m, (iv) 16.5 m s^{-1}. 5) (ii) 16 m s^{-1},
5) (iii) 90 s, (iv) 1.12 km. 6) (ii) 0.8 m s^{-2}, 5 m s^{-2}, (iii) 98.75 s, (iv) 2,016 m.
7) (ii) 30 m s^{-1}, (iii) 350 s, (iv) 7.95 km, (v) 20 m s^{-1}.

Exercise 1D

1) (i) 3 m s^{-2}, (ii) 6 m s^{-2}, (iii) 738 m, (iv) 21.1 m s^{-1}. 2) (i) 1.5 m s^{-2}, (ii) 2.5 m s^{-2},
2) (iii) 445 m, (iv) 15 m s^{-1}. 3) (i) 1.5 m s^{-2}, (ii) 3 m s^{-2}, (iii) 142.5 m, (iv) 9.5 m s^{-1}.
4) (i) 12 m s^{-1}, (ii) 94.5 m, 4 m s^{-2}. 5) (i) 2 m s^{-2}, (ii) 6 m s^{-2}, (iii) 440 m, (iv) 20 m s^{-1}.
6) (i) 7 m s^{-2}, (ii) 2 m s^{-2}, (iii) 640.5 m, (iv) 25.62 m s^{-1}, No. 7) (i) 3 m s^{-2}, (ii) 4 m s^{-2},
7) (iii) 326 m, (iv) 32.6 m s^{-1}. 8) (i) 2 m s^{-2}, (ii) 4 m s^{-2}, (iii) 440 m, (iv) 29.5 s.
9) (a) 1 m s^{-2}, 168 m, (b) 17 m s^{-1}, 0.5 m s^{-2}.