Chapter 2
Forces and Newton's Laws of Motion

Section 2A: Newton's Laws of Motion

<u>Newton's First Law of Motion</u>

Every body remains at rest or moves with constant velocity unless an external force acts on the body.

Notes on Newton's First Law of Motion:

- Usually there are a number of external forces acting on any body.
- If all the forces cancel each other out, then there is no resultant force on the body. Newton's First Law applies in this case.
- If a body has no resultant force acting on it, then it is either at rest, or moving with constant velocity.

<u>Newton's Second Law of Motion</u>

When a resultant force acts on a body, it causes the body to accelerate. The acceleration is proportional to the resultant force, and is in the same direction as the resultant force. (This assumes that the mass is unchanging).

Notes on Newton's Second Law of Motion:

- We usually summarise this law as $\vec{F} \propto \vec{a}$. Note that both the force and the acceleration are marked as vectors. Their direction must therefore be taken into account.
- It can be shown experimentally that for a given resultant force, the magnitude of the acceleration given to the body is dependent on the mass of the body.
- This gives us one of the most important formulae in applied mathematics:

$$\boxed{\vec{F} = m\vec{a}}$$

  where $\vec{F}$ is the resultant force, m is the mass, and $\vec{a}$ is the resultant acceleration.
- Usually we do not need to include the vector symbol. However we must remember that both the forces and accelerations are vectors, and therefore the direction in which they happen matters.
- Newton's Second Law of Motion only applies when there is a resultant force acting on a body. If there is no resultant force acting, i.e. all the forces cancel each other out, then Newton's First Law of Motion applies, and the body continues moving with constant velocity, or is at rest.

<u>Newton's Third Law of Motion</u>

When a force acts on a body, an equal and opposite force acts on another body.

Notes on Newton's Third Law of Motion:

- This law is often stated as: Action and reaction are equal and opposite.
- When two bodies are in contact, the force applied by the first on the second is equal and opposite to the force applied by the second on the first.
- This is of relevance to applied maths in connection with Normal Reaction forces.

Weight: The weight of a body is the force of the earth's gravitational field on it. It always acts vertically downwards, towards the centre of the earth. When a body is freefalling near the earth, it accelerates towards the earth with the acceleration due to gravity, g. At Leaving Certificate Ordinary Level, g is taken to be 10 m s^{-2}. At Leaving Certificate Higher Level it is taken to be 9.8 m s^{-2}. If we use this in $\vec{F} = m\vec{a}$, then we can show that:

Weight = Mass x Acceleration due to gravity $\boldsymbol{W = mg = 10m}$ **N**

Note for this formula for weight, **the mass must be in kilograms** (kg).

Notes on Drawing Diagrams for Force and Motion Problems:

- For all force problems a clear and complete diagram must be drawn, showing all the forces, masses and accelerations.
- Any bodies should be drawn as simple rectangles (occasionally circles), with the mass of the body written inside the rectangle.
- All forces should be drawn as arrows coming out of the objects in the correct direction.
- All accelerations are drawn as double headed arrows in the correct direction, just beside the body to which they apply. Do not draw the acceleration coming out of the object as it is too easy then to confuse it with a force. Always mark the acceleration in the direction the body is moving. Therefore if the body is decelerating, the acceleration will be negative.

Solution Strategy for problems involving single particle force problems moving horizontally or vertically: 1) **Draw a detailed diagram** of the system, making sure to include all forces, masses and accelerations. 2) **Work out the Equation of Motion** for the particle, using $\boldsymbol{F = ma}$, in the direction in which the acceleration is marked, and then solve. 3) Use the **constant acceleration formulae** as required to find velocities, times and distances.

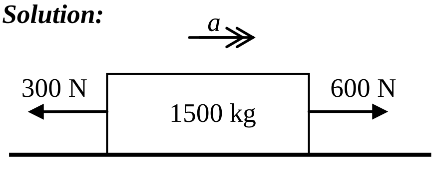
Example 2A1

A car of mass 1500 kg accelerates from rest on a straight level road against a resistance of 300 N. The engine provides a tractive force of 600 N. The car starts from rest.

(i) Find the acceleration of the car.

(ii) Find how long it takes the car to reach a speed of 16 m s^{-1}.

Solution:



(i) The equation of motion of the car to the right,

$\vec{F} = m\vec{a}$

$\Rightarrow 600 - 300 = 1500a, \qquad \Rightarrow a = 0.2 \text{ m s}^{-2}$

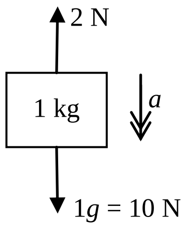
(ii) $u = 0 \text{ m s}^{-1}$, $v = 16 \text{ m s}^{-1}$, $a = 0.2 \text{ m s}^{-2}$, $t = ???$

$v = u + at$

$\Rightarrow 16 = 0 + 0.2t, \qquad \Rightarrow t = 80$ s.

Example 2A2
A ball of mass 1 kg falls from rest at the top of a 100 m high building against a constant air resistance of 2 N.
(i) Find its acceleration.
(ii) Find how long does it take the ball to hit the ground.

Solution:



(i) The equation of motion of the ball downwards,

$\vec{F} = m\vec{a}$

$\Rightarrow 10 - 2 = 1a$

$\Rightarrow a = 8 \text{ m s}^{-2}$

(ii) $u = 0, \quad a = 8 \text{ m s}^{-2}, \ s = 100 \text{ m}, \ t = ???$

$s = ut + \frac{1}{2}at^2$

$\Rightarrow 100 = 0t + \frac{1}{2}(8)t^2$

$\Rightarrow t^2 = \frac{100}{4} = 25, \qquad \Rightarrow t = 5 \text{ s}.$

Exercise 2A – using the equation of motion horizontally and vertically

1) A bus of mass 8,000 kg accelerates uniformly from rest to 12 m s^{-1} in 24 s on a straight horizontal road.
(i) Find its acceleration.
(ii) There is a constant resistance of 20,000 N between the bus and the road. Draw a diagram showing the forces on the bus, and hence find the tractive force on the bus.

2) A dog team pulls a sled of mass 500 kg from rest across 800 m of ice in 80 s. The sled accelerates uniformly. The rope attaching the sled to the dog team was kept horizontal at all times, and there is no resistance between the sled and the ice underneath.
(i) Find the acceleration of the sled.
(ii) Draw a diagram of the forces on the sled, and hence find the pulling force of the dog team to cause this acceleration.

3) An overtaking car of mass 1800 kg accelerates uniformly from 20 m s^{-1} with a tractive force of 6300 N, against a constant resistive force of 3600 N.
(i) Draw a diagram showing the forces on the car, and hence find the acceleration of the car.
(ii) Find how far extra the car travels in 10 s due to the acceleration, compared to if it still travelled at a constant 20 m s^{-1}.

4) A brick of mass 0.5 kg falls vertically to the ground from rest from a height of 14 m. The air resistance is a constant 1.5 N.
(i) Draw a diagram showing the forces on the brick, and hence find the acceleration of the brick.
(ii) How long does it take the brick to reach the ground?

5) A stone of mass 0.09 kg is dropped from rest from the top of a building 31.25 m high.
(i) Find the time taken for the stone to hit the ground if there is no air resistance.
(ii) If in fact it actually takes the stone 3.75 s to hit the ground, find the acceleration of the stone. Express your answer as a fraction.
(iii) Draw a diagram showing the forces on the stone, and hence find the constant force due to air resistance experienced by the stone as it falls.

6) A car is brought to rest with uniform deceleration from a speed of 30 m s^{-1} in a distance of 75 m. The car has a mass of 1,500 kg.
(i) Find the deceleration of the car.
(ii) If there is no tractive force on the car, but there is a resistance between the car and the road of 3,500 N, and there is a braking force, draw a diagram showing all the forces acting on the car. Hence find the braking force of the car.

7) A lift of mass 400 kg accelerates up a lift shaft with acceleration of 1.5 m s^{-2}. There is a resistive force of 1,000 N between the lift and the lift shaft. Draw a diagram showing all the forces on the lift, and hence find the tension in the lift cable causing the upwards acceleration.

Section 2B: Simple Pulley Systems

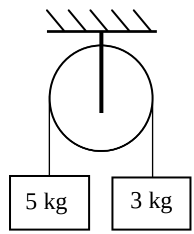
In the force problems we have seen so far, there has only been one moving particle. In many problems however there is a system of particles joined together by strings, usually involving pulleys. In these problems the string is always considered to be inelastic, which means that the tension is the same throughout the string. The string is also considered as light, which means that we regard it as having no mass. The tension forces always pull away from the particles, they never "push" the particles. These problems also involve the strings passing over pulleys. These pulleys are always considered to be smooth and light. This means that we assume that it takes no effort to actually turn the pulley, and therefore the tension in the string is the same on both sides of the pulley.

Solution Strategy for simple systems of particles:

1) **Draw a detailed diagram** of the system, making sure to include all forces, masses and accelerations.
2) **Work out the Equation of Motion** for each particle in the system using $\boldsymbol{F = ma}$, in the direction in which it is moving. The heavier particle will move downwards, and the lighter one upwards. There are two forces on each particle, weight downwards and tension upwards.
3) Solve these equations using **simultaneous equations**.

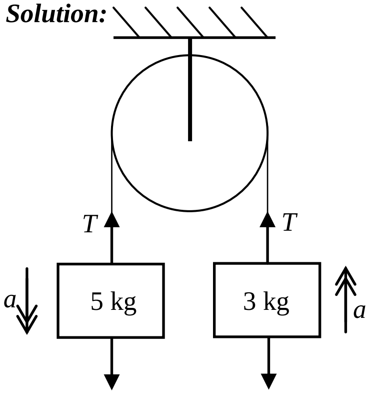
Example 2B1

Two particles of masses 5 kg and 3 kg are connected by a taut, light, inextensible string which passes over a smooth light fixed pulley. The system is released from rest. Show on a diagram the forces acting on each particle.



(i) Write down the equations of motion for each particle.
(ii) Find the common acceleration of the two particles.
(iii) Find the tension in the string.

Solution:



(i) When putting the accelerations on, the heavier mass (5 kg) will move downwards, and the lighter mass (3 kg) will move upwards. It is very important that the accelerations marked on both particles are consistent with each other. Then the equation of motion for each particle is done in the direction the acceleration is marked for that particle.

The equations of motion for the two masses, $\vec{F} = m\vec{a}$:

5 kg $\downarrow$: $50 - T = 5a$ **A**

3 kg $\uparrow$: $T - 30 = 3a$ **B**

(ii) To solve these, add the two equations together:

A + **B**: $20 = 8a$, $\Rightarrow a = \frac{20}{8} = 2.5 \text{ m s}^{-2}$

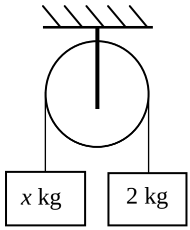
(iii) Now substitute this back in to equation **B** to find T:

In **B**: $T - 30 = 3(2.5)$, $\Rightarrow T = 37.5$ N.

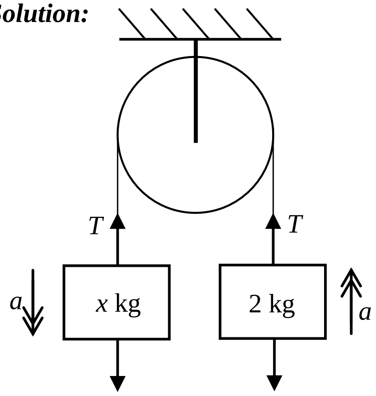
Example 2B2

A light elastic string passes over a smooth light pulley. A mass of *x* kg is attached to one end of the string and a mass of 2 kg is attached to the other end. When the system is released from rest, the 2 kg mass rises 12 m in 2 seconds. Show on a diagram the forces acting on each particle.

(i) Find the common acceleration of the two particles.
(ii) Write down the equations of motion for each particle.
(iii) Find the tension in the string.
(iv) Find the value of *x*.



Solution:



(i) For the 2 kg particle:

$u = 0$, $t = 2$ s, $s = 12$ m, $a = ???$

$s = ut + \frac{1}{2}at^2$, $\Rightarrow 12 = 0(2) + \frac{1}{2}a(2)^2$

$\Rightarrow 2a = 12$, $\Rightarrow a = 6 \text{ m s}^{-2}$

(ii) The equations of motion for the two masses,

$\vec{F} = m\vec{a}$: x kg $\downarrow$: $10x - T = 6x$ **A**

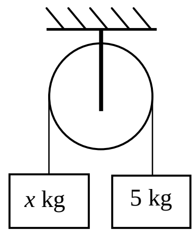
2 kg $\uparrow$: $T - 20 = 2(6) = 12$ **B**

(iii) In **B**: $T - 20 = 12$, $\Rightarrow T = 32$ N.

(iv) In **A**: $10x - 32 = 6x$, $\Rightarrow 4x = 32$

$\Rightarrow x = 8$ kg.

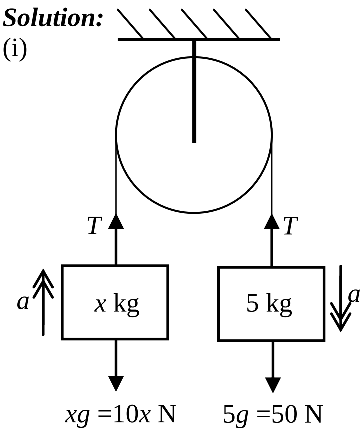
Example 2B3



Two particles of masses x kg and 5 kg are connected by a light inextensible string which passes over a smooth fixed pulley. The particles are released from rest.

(i) Show on a diagram the forces acting on each particle.
(ii) Write down the equation of motion for each particle.
(iii) Find, in terms of x, the acceleration of the particles and the tension in the string.

Solution:
(i)



(ii) As we don't know which mass is heavier, and in which direction the two masses move, it doesn't matter which direction we choose, as long as the two accelerations are consistent with each other, i.e. if one goes up the other must go down. We will put the particle of mass x kg going up.

The equations of motion for the two masses,

$\vec{F} = m\vec{a}$: x kg $\uparrow$: $T - 10x = xa$ **A**

5 kg $\downarrow$: $50 - T = 5a$ **B**

(iii) Adding the two equations together:

A+**B**: $50 - 10x = xa + 5a = a(x+5)$

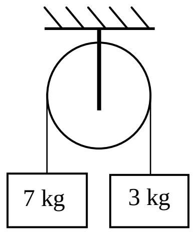
$$\Rightarrow a = \frac{50-10x}{x+5} \text{ m s}^{-2}$$

In **B**: $50 - T = 5\left(\frac{50-10x}{x+5}\right)$

$$\Rightarrow T = 50 - 5\left(\frac{50-10x}{x+5}\right) = \frac{50(x+5) - 5(50-10x)}{x+5}$$

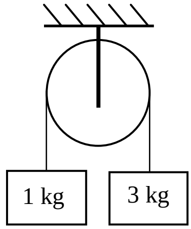
$$\Rightarrow T = \frac{50x + 250 - 250 + 50x}{x+5} = \frac{100x}{x+5} \text{ N.}$$

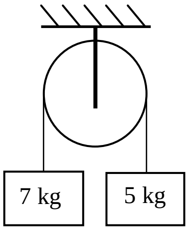
Exercise 2B –problems involving simple pulley systems



1) Two particles of masses 7 kg and 3 kg are connected by a taut, light, inextensible string which passes over a smooth light fixed pulley. The system is released from rest. Show on a diagram the forces acting on each particle.
(i) Write down the equations of motion for each particle.
(ii) Find the common acceleration of the two particles.
(iii) Find the tension in the string.

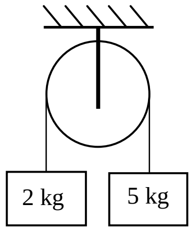
2) Two particles of masses 1 kg and 3 kg are connected by a taut, light, inextensible string which passes over a smooth light fixed pulley. The system is released from rest. Show on a diagram the forces acting on each particle.
(i) Write down the equations of motion for each particle.
(ii) Find the common acceleration of the two particles.
(iii) Find the tension in the string.

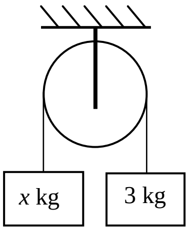




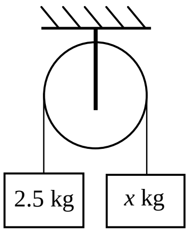
3) Two particles of masses 7 kg and 5 kg are connected by a taut, light, inextensible string which passes over a smooth light fixed pulley. The particles are released from rest. Show on a diagram the forces acting on each particle.
(i) Write down the equations of motion for each particle.
(ii) Find the common acceleration of the two particles.
(iii) Find the tension in the string.

4) Two particles of masses 2 kg and 5 kg are connected by a taut, light, inextensible string which passes over a smooth light fixed pulley. The system is released from rest. Show on a diagram the forces acting on each particle.
(i) Write down the equations of motion for each particle.
(ii) Find the common acceleration of the two particles.
(iii) Find the tension in the string.



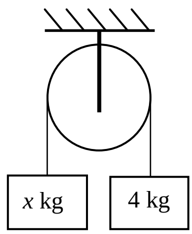


5) A light elastic string passes over a smooth light pulley. A mass of x kg is attached to one end of the string and a mass of 3 kg is attached to the other end. When the system is released from rest, the 3 kg mass falls 9 m in 3 seconds. Show on a diagram the forces acting on each particle.
(i) Find the common acceleration of the two particles.
(ii) Write down the equations of motion for each particle.
(iii) Find the tension in the string, and hence the value of x.

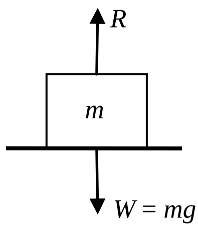


6) A light elastic string passes over a smooth light pulley. A mass of 2.5 kg is attached to one end of the string and a mass of x kg is attached to the other end. When the system is released from rest, the 2.5 kg mass falls 2.8125 m in 1.5 seconds. Show on a diagram the forces acting on each particle.
(i) Find the common acceleration of the two particles.
(ii) Write down the equations of motion for each particle.
(iii) Find the tension in the string, and hence find the value of x.

7) Two particles of masses x kg and 4 kg are connected by a light inextensible string which passes over a smooth fixed pulley. The particles are released from rest. Show on a diagram the forces acting on each particle.
(i) Write down the equation of motion for each particle.
(ii) Find, in terms of x, the acceleration of the particles and the tension in the string.



Section 2C: Rough Horizontal Surfaces



From Newton's Third Law of Motion, when a body is resting in contact with a surface, it experiences a force from the surface. We call this force the **Normal Reaction**. It always acts perpendicular to the surface, so on a horizontal surface it acts vertically upwards. We usually give it the symbol R, sometimes N. In order to find R, we need to **resolve vertically**. You can resolve in a given direction when there is no acceleration, or component of acceleration, in that direction. Resolving vertically just means that the total of the upward forces = the total of the downward forces.

When any two surfaces rub over one and other, there is a force between them which opposes the motion. We call this force **Friction**. If there is no friction between two surfaces, they are called **smooth**. When there is friction they are called **rough**. We use the symbol F for friction.

The Laws of Friction

- Friction opposes the movement, or potential movement, of an object across the surface of another object, unless the surface is smooth.
- The direction of the frictional force is always opposite to the direction of motion, or to the potential direction of motion.
- The magnitude of the frictional force is, up to a certain limit, equal to the magnitude of the force tending to cause the movement. In this situation the object is in equilibrium (i.e. not accelerating).
- The largest friction force possible is called **Limiting Friction**. If the force tending to cause movement increases any further, the friction force cannot increase any further, and the object starts to accelerate. For limiting friction to apply, the two surfaces must be moving over one and other, or be just about to move over one and other. Limiting friction is also known as limiting equilibrium.
- The limiting friction force is calculated using the following formula:

$$\boldsymbol{F = \mu R}$$

  where F is the limiting friction force, R is the normal reaction force between the two surfaces, and μ is the coefficient of friction.
- The **coefficient of friction** (μ) is a property of the two surfaces. μ is usually taken as between 0 and 1, though in fact it can be larger than 1. If $\mu = 0$ then the surfaces are smooth.

Solution Strategy for force questions involving friction on the horizontal plane:

1) **Draw a detailed diagram** of the system, making sure to include all forces, masses and accelerations.
2) **Resolve vertically** to find R. Use $\boldsymbol{F = \mu R}$ to calculate the friction force.
3) **Work out the Equation of Motion** for the particle using $\boldsymbol{F = ma}$, and solve.
4) Use the **constant acceleration formulae** where necessary to find velocities, times and distances.

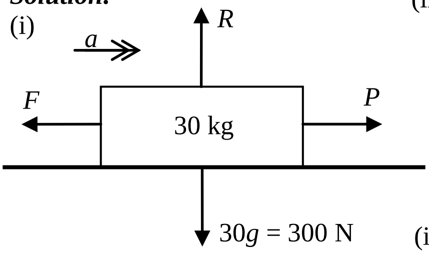
Example 2C1

A man wants to push a box of mass 30 kg across a room. The coefficient of friction between the box and the rough floor is 0.4.

(i) Show on a diagram all the forces acting on the box.
(ii) With what force must the man push the box to just get it moving?
(iii) If the man pushes with a force of 150 N, find the acceleration of the box.

Solution:

(i)



(ii) Resolving vertically: $\Rightarrow R = 300$ N.

$F = \mu R \qquad \Rightarrow F = 0.4(300) = 120$ N.

For the box just to start moving, the man must push with a force equal to the limiting friction force.

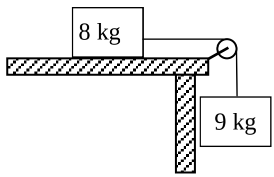
$\Rightarrow$ The man pushes with 120 N.

(iii) The equation of motion of the box to the right,

$\vec{F} = m\vec{a}: \quad \Rightarrow 150 - 120 = 30a, \qquad \Rightarrow a = \frac{30}{30} = 1 \text{ m s}^{-2}$

Example 2C2

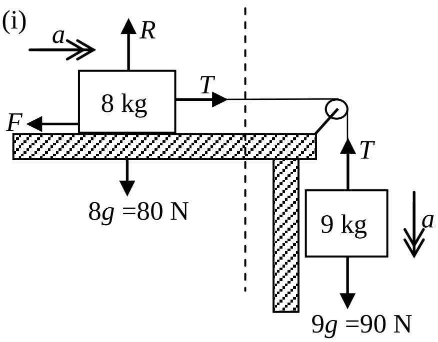
A particle of mass 8 kg is connected to another particle of mass 9 kg by a taut light inelastic string which passes over a light smooth pulley at the edge of a rough horizontal table. The coefficient of friction between the 8 kg mass and the table is ¼. The system is released from rest.



(i) Show on separate diagrams the forces acting on each particle.
(ii) Find the common acceleration of the two particles.
(iii) Find the tension in the string.

Solution:

(i)



(ii) Resolving vertically for the 8 kg:

$R = 80$ N.

$F = \mu R, \qquad \Rightarrow F = 0.25(80) = 20$ N.

The equations of motion of the masses,

$\vec{F} = m\vec{a}$: 8 kg $\rightarrow$: $T - 20 = 8a$ **A**

9 kg $\downarrow$: $90 - T = 9a$ **B**

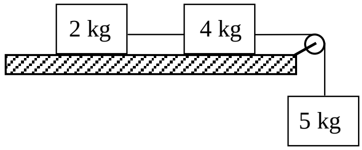
To solve these, add **A** and **B** together.

A+**B**: $70 = 17a, \qquad \Rightarrow a = \frac{70}{17}$ m s^{-2}

(iii) In **A**: $T - 20 = 8\left(\frac{70}{17}\right), \qquad \Rightarrow T = 52.9$ N.

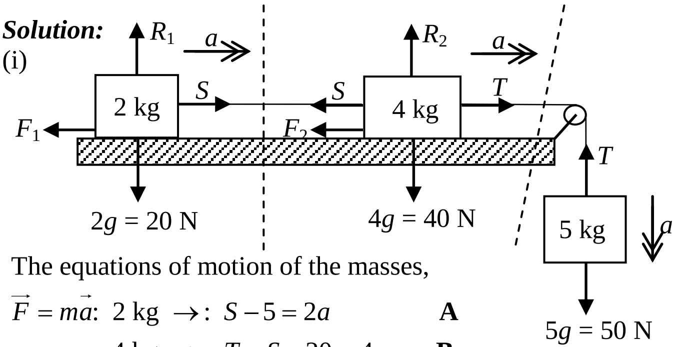
Example 2C3

Particles, of masses 2 kg and 4 kg, resting on a rough horizontal table, are connected by a light taut inextensible string. The coefficient of friction between the 2 kg mass and the table is ¼ and between the 4 kg mass and the table is ½. The 4 kg mass is connected by a second light inextensible string passing over a smooth light pulley at the edge of the table to a particle of mass 5 kg. The 5 kg mass hangs freely under gravity. The particles are released from rest. The 5 kg mass moves vertically downwards.



(i) Show on separate diagrams all the forces acting on each particle.
(ii) Write down the equation of motion for each particle.
(iii) Find the common acceleration of the particles and the tension in each string.

Solution:
(i)



(ii) Resolving vertically:

2 kg: $R_1 = 20$ N.

4 kg: $R_2 = 40$ N.

$F = \mu R$:

2 kg: $F_1 = 0.25(20) = 5$ N.

4 kg: $F_2 = 0.5(40) = 20$ N.

The equations of motion of the masses,

$\vec{F} = m\vec{a}$: 2 kg $\rightarrow$: $S - 5 = 2a$ **A**

4 kg $\rightarrow$: $T - S - 20 = 4a$ **B**

5 kg $\downarrow$: $50 - T = 5a$ **C**

(iii) **A** + **B** + **C**: $25 = 11a$, $\Rightarrow a = \frac{25}{11}$ m s^{-2}

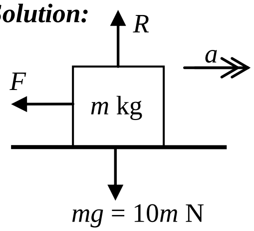
In **A**: $S = 2a + 5 = \frac{50}{11} + 5 = 9.55$ N.

In **C**: $T = 50 - 5a = 50 - \frac{125}{11} = 38.64$ N.

Example 2C4

Calculate the initial speed that a stone must be given to make it skim horizontally across ice so that it comes to rest after skimming 50 m. The coefficient of friction between the stone and the ice is 0.4.

Solution:



Resolving vertically: $\Rightarrow R = 10m$ N.

$F = \mu R$: $\Rightarrow F = 0.4(10m) = 4m$ N.

The equation of motion to the right, $\vec{F} = m\vec{a}$:

$\Rightarrow -4m = ma$, $\Rightarrow a = -4$ m s^{-2}

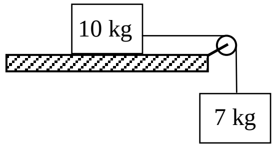
$s = 50$ m, $a = -4$ m s^{-2}, $v = 0$, $u = ???$ $v^2 = u^2 + 2as$

$\Rightarrow 0^2 = u^2 + 2(-4)(50)$, $\Rightarrow u^2 = 400$, $\Rightarrow u = 20$ m s^{-1}

Exercise 2C – applying friction to horizontal motion problems

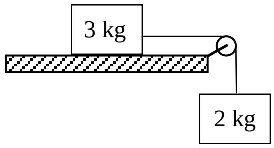
1) A tractor pulls a log of mass 400 kg by means of a horizontal rope across rough horizontal ground. If the tension in the rope is 2,600 N, and the coefficient of friction between the log and the ground is 0.5, find the acceleration of the log.

2) A particle of mass 10 kg is connected to another particle of mass 7 kg by a taut light inelastic string which passes over a light smooth pulley at the edge of a rough horizontal table. The coefficient of friction between the 10 kg mass and the table is ½. The system is released from rest.



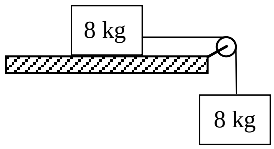
   (i) Show on separate diagrams the forces acting on each particle.
   (ii) Find the common acceleration of the two particles.
   (iii) Find the tension in the string.

3) Two particles of masses 3 kg and 2 kg are connected by a taut light inextensible string which passes over a light smooth pulley at the edge of a rough horizontal table. The coefficient of friction between the 3 kg mass and the table is ⅓. The system is released from rest.



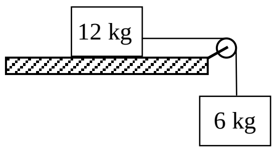
   (i) Show on separate diagrams the forces acting on each particle.
   (ii) Find the common acceleration of the two particles.
   (iii) Find the tension in the string.

4) A particle of mass 8 kg is connected to another particle of mass 8 kg by a taut light inelastic string which passes over a light smooth pulley at the edge of a rough horizontal table. The coefficient of friction between the 8 kg mass and the table is 0.2. The system is released from rest.



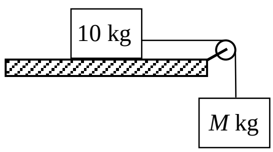
   (i) Show on separate diagrams the forces acting on each particle.
   (ii) Find the common acceleration of the two particles.
   (iii) Find the tension in the string.

5) Two particles of masses 12 kg and 6 kg are connected by a taut light inextensible string which passes over a light smooth pulley at the edge of a rough horizontal table. The coefficient of friction between the 12 kg mass and the table is μ. The 6 kg mass hangs freely under gravity. The system is released from rest. The 6 kg mass moves vertically downwards with an acceleration of $\frac{10}{9}$ m s^{-2}.



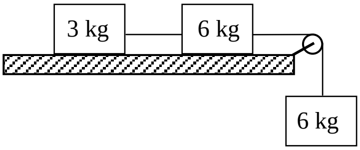
   (i) Show on separate diagrams all the forces acting on each particle.
   (ii) Find the value of the tension in the string.
   (iii) Find the value of μ, giving your answer as a fraction.

6) Two particles of masses 10 kg and M kg are connected by a taut light inextensible string which passes over a light smooth pulley at the edge of a rough horizontal table. The coefficient of friction between the 10 kg mass and the table is ¼. The M kg mass hangs freely under gravity. The particles are released



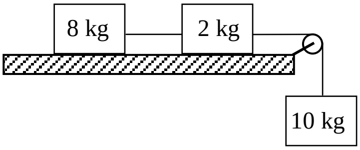
from rest. The M kg mass moves vertically downwards with an acceleration of 2 m s^{-2}.
(i) Show on separate diagrams all the forces acting on each particle.
(ii) Find the value of the tension in the string.
(iii) Find the value of M.

7) Particles, of masses 3 kg and 6 kg, resting on a rough horizontal table, are connected by a light taut inextensible string. The coefficient of friction between the 3 kg mass and the table is ⅔ and between the 6 kg mass and the table is ⅓. The 6 kg mass is connected by a second light inextensible string passing over a smooth light pulley at the edge of the table to a second particle of mass 6 kg, hanging freely under gravity. The particles are released from rest.



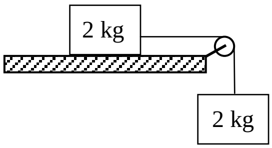
(i) Show on separate diagrams all the forces acting on each particle.
(ii) Write down the equation of motion for each particle.
(iii) Find the common acceleration of the particles and the tension in each string.

8) Particles, of masses 8 kg and 2 kg, resting on a rough horizontal table, are connected by a light taut inextensible string. The coefficient of friction between the 8 kg mass and the table is ¾ and between the 2 kg mass and the table is ¼. The 2 kg mass is connected by a second light inextensible string passing over a smooth light pulley at the edge of the table to a particle of mass 10 kg, hanging freely under gravity. The particles are released from rest.



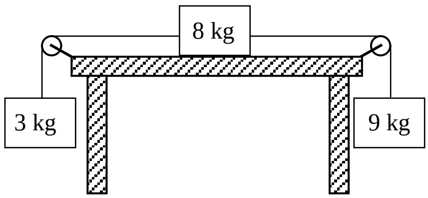
(i) Show on separate diagrams all the forces acting on each particle.
(ii) Write down the equation of motion for each particle.
(iii) Find the common acceleration of the particles and the tension in each string.

9) Two blocks, each of mass 2 kg, are connected by a taut light inextensible string which passes over a light smooth pulley at the edge of a smooth horizontal table. One block is at rest on the smooth horizontal table, and the second block is hanging freely under gravity.



(i) When the system is released from rest, find the common acceleration of the two blocks.
(ii) If the smooth table is replaced by a rough table and the system is released from rest, as before, the common acceleration of the blocks is half what it was in the first case. Find the coefficient of friction between the block on the table and the table.

10) A particle of mass 8 kg lies on a rough horizontal table. The coefficient of friction between the particle and the table is ½. The particle is connected by two taut light inelastic strings passing over smooth pulleys at opposite edges of the table to two masses of 3 kg and 9 kg which hang freely, as in the diagram. The system is released from rest and the 9 kg particle moves downwards.



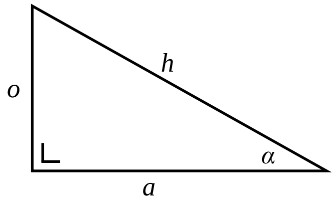
(i) Show on separate diagrams the forces acting on each particle.
(ii) Calculate the size of the friction force on the 8 kg particle.
(iii) Write down the equation of motion for each particle.
(iv) Find the common acceleration of the system, and the tensions in the two strings.

11) In a curling match, a player slides a stone across the ice. If the coefficient of friction between the stone and the ice is 0.1, what initial speed should he give to the stone if it is to stop exactly 32 m away?

12) A car of mass 2000 kg is travelling along a straight horizontal road at a constant speed of 20 m s^{-1}, when the engine is shut off, and the car freewheels to a stop under friction. If the coefficient of friction between the car and the road is 0.25, find the distance travelled by the car in coming to rest.

Section 2D: Introduction to Surds and Trigonometry

Very often in Applied Mathematics trigonometry is used to calculate the sides of a right-angled triangle where one angle is known, as well as the right angle. In order to do this the **sine** (sin) and the **cosine** (cos) of the angle must be known. Usually the **tangent** (tan) of the angle is given as a fraction, and from this the sine and cosine must be calculated. In these cases it is better to calculate these as fractions, maybe involving surds, rather than using the calculator to calculate them in decimal form. A **surd** is just a whole number inside a square root, e.g. $\sqrt{5}$. This involves using **Pythagoras' Theorem** to calculate the third (unknown) side of the right-angled triangle, and then using the relevant formulae for the sine, cosine and tangent ratios. For a right-angled triangle, the longest side, which is opposite the right angle is known as the hypotenuse (h). The side which is beside the known angle (α) is known as the adjacent side (a) and the side on the other side of the triangle from the known angle is called the opposite side (o).



The three trigonometric ratios are:

$\sin\alpha = \frac{o}{h}$, $\cos\alpha = \frac{a}{h}$ and $\tan\alpha = \frac{o}{a}$.

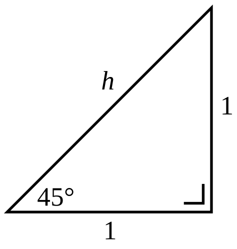
Pythagoras' Theorem: $h^2 = a^2 + o^2$.

Example 2D1

$\tan 45^\circ = 1$. Write $\sin 45^\circ$ and $\cos 45^\circ$ as fractions, using surds where necessary.

Solution:

The first step is to draw a right-angled triangle, putting in the known sides, and from these calculate the third side using Pythagoras' Theorem. In this case the tan of the angle is given as a whole number, 1, so we write this as a fraction by putting it over 1, i.e. $\frac{1}{1}$. Therefore both the opposite and adjacent sides are 1.



$\tan 45^\circ = 1 = \frac{1}{1}$

Therefore both the opposite and adjacent sides are 1.

By Pythagoras' Theorem: $h^2 = 1^2 + 1^2, \quad \Rightarrow h = \sqrt{2}$

$\sin 45^\circ = \frac{o}{h} = \frac{1}{\sqrt{2}}, \qquad \cos 45^\circ = \frac{a}{h} = \frac{1}{\sqrt{2}}$

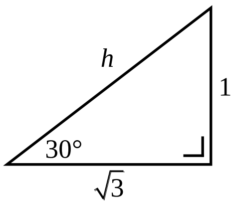
Note that most calculators give $\sin 45^\circ = \cos 45^\circ = \frac{\sqrt{2}}{2}$. This is equal to $\frac{1}{\sqrt{2}}$.

Example 2D2

$\tan 30^\circ = \frac{1}{\sqrt{3}}$. Write $\sin 30^\circ$ and $\cos 30^\circ$ as fractions, using surds where necessary.

Solution:

The first step is to draw a right-angled triangle, putting in the known sides, and from these calculate the third side using Pythagoras' Theorem. In this case the tan of the angle is $1/\sqrt{3}$, so the opposite is 1 and the adjacent is $\sqrt{3}$.



$\tan 30^\circ = \frac{1}{\sqrt{3}}$. Therefore the opposite is 1 and the adjacent is $\sqrt{3}$.

By Pythagoras' Theorem: $h^2 = 1^2 + \left(\sqrt{3}\right)^2 = 1 + 3 = 4, \quad \Rightarrow h = \sqrt{4} = 2$

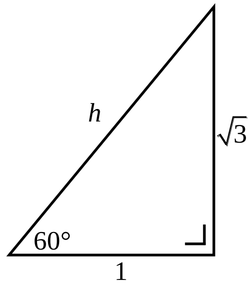
$\sin 30^\circ = \frac{o}{h} = \frac{1}{2}, \qquad \cos 30^\circ = \frac{a}{h} = \frac{\sqrt{3}}{2}$.

Example 2D3

$\tan 60^\circ = \sqrt{3}$. Write $\sin 60^\circ$ and $\cos 60^\circ$ as fractions, using surds where necessary.

Solution:

The first step is to draw a right-angled triangle, putting in the known sides, and from these calculate the third side using Pythagoras' Theorem. In this case the tan of the angle is $\sqrt{3}$, so the opposite is $\sqrt{3}$ and the adjacent is 1.



$\tan 60^\circ = \sqrt{3}$. Therefore the opposite is $\sqrt{3}$ and the adjacent is 1.

By Pythagoras' Theorem: $h^2 = \left(\sqrt{3}\right)^2 + 1^2 = 3 + 1 = 4, \quad \Rightarrow h = \sqrt{4} = 2$

$\sin 60^\circ = \frac{o}{h} = \frac{\sqrt{3}}{2}, \qquad \cos 60^\circ = \frac{a}{h} = \frac{1}{2}$.

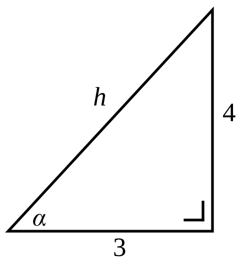
The results from the previous three examples occur regularly throughout the Applied Mathematics course, and it is worth students taking note of them. They are given on page 13 of the book of Formulae and Tables, published by the State Examinations Commission.

Angle α	30°	45°	60°
sin α	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos α	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan α	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Example 2D4

If $\tan\alpha = \frac{4}{3}$, write $\sin\alpha$ and $\cos\alpha$ as fractions, using surds where necessary.

Solution:



The first step is to draw a right-angled triangle, putting in the known sides, and from these calculate the third side using Pythagoras' Theorem. In this case the tan of the angle is $\frac{4}{3}$, so the opposite is 4 and the adjacent is 3.

$\tan\alpha = \frac{4}{3}$. Therefore the opposite is 4 and the adjacent is 3.

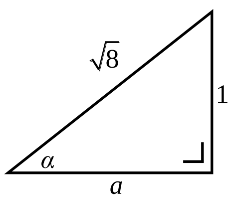
By Pythagoras' Theorem: $h^2 = 4^2 + 3^2 = 16 + 9 = 25, \quad \Rightarrow h = \sqrt{25} = 5$

$$\sin\alpha = \frac{o}{h} = \frac{4}{5}, \qquad \cos\alpha = \frac{a}{h} = \frac{3}{5}.$$

Example 2D5

If $\sin\alpha = \frac{1}{\sqrt{8}}$, write $\tan\alpha$ and $\cos\alpha$ as fractions, using surds where necessary.

Solution:



The first step is to draw a right-angled triangle, putting in the known sides, and from these calculate the third side using Pythagoras' Theorem. In this case the sin of the angle is $\frac{1}{\sqrt{8}}$, so the opposite is 1 and the hypotenuse is $\sqrt{8}$.

$\sin\alpha = \frac{1}{\sqrt{8}}$. Therefore the opposite is 1 and the hypotenuse is $\sqrt{8}$.

By Pythagoras' Theorem: $\left(\sqrt{8}\right)^2 = 1^2 + a^2$

$$\Rightarrow 8 = 1 + a^2, \quad \Rightarrow a^2 = 8 - 1 = 7, \quad \Rightarrow a = \sqrt{7}$$

$$\tan\alpha = \frac{o}{a} = \frac{1}{\sqrt{7}}, \qquad \cos\alpha = \frac{a}{h} = \frac{\sqrt{7}}{\sqrt{8}} = \sqrt{\frac{7}{8}}.$$

Exercise 2D – surds and trigonometry

1) Write as fractions, using surds where necessary: sin 30°, cos 30° and tan 30°.
2) Write as fractions, using surds where necessary: sin 45°, cos 45° and tan 45°.
3) Write as fractions, using surds where necessary: sin 60°, cos 60° and tan 60°.
4) For each of the following values of tan α, write sin α and cos α as fractions, using surds where necessary.

   (i) $\tan\alpha = \frac{3}{4}$, (ii) $\tan\alpha = \frac{12}{5}$, (iii) $\tan\alpha = \frac{7}{24}$, (iv) $\tan\alpha = \frac{15}{8}$, (v) $\tan\alpha = \frac{1}{2}$,

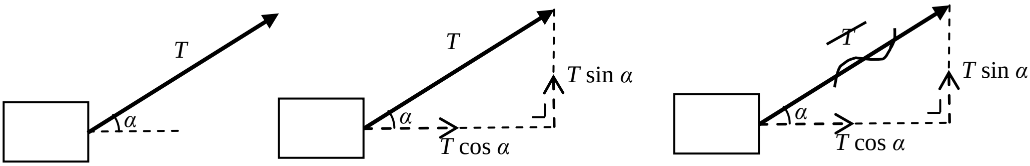
   (vi) $\tan\alpha = 3$, (vii) $\tan\alpha = \frac{1}{\sqrt{2}}$, (viii) $\tan\alpha = \sqrt{2}$, (ix) $\tan\alpha = \frac{1}{5}$.
5) For each of the following values of sin α, write cos α and tan α as fractions, using surds where necessary.

   (i) $\sin\alpha = \frac{4}{5}$, (ii) $\sin\alpha = \frac{5}{13}$, (iii) $\sin\alpha = \frac{1}{3}$, (iv) $\sin\alpha = \frac{2}{7}$, (v) $\sin\alpha = \frac{1}{\sqrt{3}}$.
6) For each of the following values of cos α, write sin α and tan α as fractions, using surds where necessary.

   (i) $\cos\alpha = \frac{24}{25}$, (ii) $\cos\alpha = \frac{1}{2}$, (iii) $\cos\alpha = \frac{3}{4}$, (iv) $\cos\alpha = \frac{8}{17}$, (v) $\cos\alpha = \frac{2}{\sqrt{7}}$.

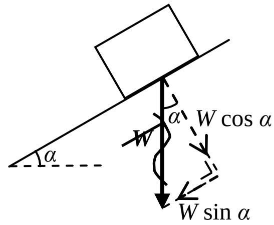
Section 2E – Forces on Smooth Inclined Planes

So far we have only dealt with force problems which are either in the horizontal or vertical directions. Many problems however involve forces at various other angles, and we will deal with these now. This will involve breaking forces in to their components in two perpendicular directions, using the sine and cosine of the angle.



The series of diagrams above shows how to break a force T at angle α to the horizontal into horizontal and vertical components. The force T is the hypotenuse, the side adjacent to the angle is the *cos* component, and the side opposite the angle is the *sin* component. Therefore T is being replaced by $(T\cos\alpha + T\sin\alpha)$. The final step is to cross out the original force T, as otherwise the same force T is included twice, as itself and broken into its components.
Note: It is very important that when a force has been broken into its components on a diagram, that the original force is crossed out. Alternatively the components can be drawn in a different colour to the original. The original force must either be crossed out, or very clearly distinguishable from its components, otherwise the same force is effectively included twice on the diagram, which is obviously wrong.

When a particle is on the inclined plane, you must break the weight into components along and perpendicular to the plane. The angle (α) which the plane makes with the horizontal is the same as the angle the weight makes with the normal (perpendicular) to the plane, as shown in the diagram to the right. When you resolve perpendicular to the plane to find R, you use $W\cos\alpha$. When you do the equation of motion along the plane, use $W\sin\alpha$.



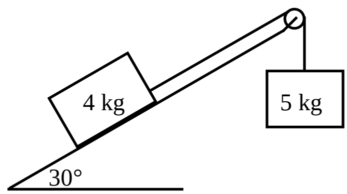
In this section, as we are dealing with smooth inclined planes only, we will only be considering the component of the weight parallel to the inclined plane (the *sin* component). In the following section we will deal with rough inclined planes.

Solution Strategy for questions with objects sliding on smooth inclined planes:
1) **Draw a detailed diagram** of the system, making sure to include all forces, masses and accelerations.
2) **Resolve vertically** for any masses on rough horizontal planes to find the normal reactions R. Use $\boldsymbol{F = \mu R}$ to calculate any friction forces.
3) **Work out the Equation of Motion** for each mass in the system. For any particles on an inclined plane, the direction of the equation of motion is along the plane.
4) Solve these equations using **simultaneous equations**.

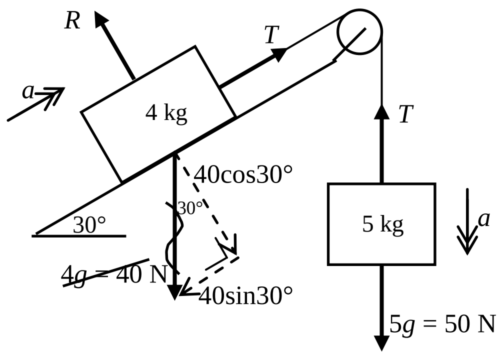
Example 2E1

Masses of 4 kg and 5 kg are connected by a taut light inelastic string which passes over a smooth light pulley, as shown in the diagram. The 4 kg mass lies on a smooth plane inclined at 30° to the horizontal. The 5 kg mass hangs vertically. The system is released from rest, and the 5 kg mass moves downwards.

Find: (i) the common acceleration of the two particles,
(ii) the tension in the string.



Solution:



(i) The equations of motion of the masses, $\vec{F} = m\vec{a}$:

4 kg ↗: $T - 40\sin 30^\circ = 4a$

$\Rightarrow T - 20 = 4a$ **A**

5 kg ↓: $50 - T = 5a$ **B**

A+**B**: $30 = 9a$

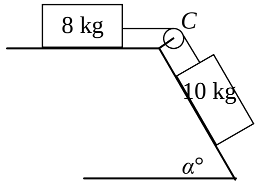
$$\Rightarrow a = \frac{30}{9} = \frac{10}{3} \text{ m s}^{-2}$$

(ii) In **A**: $T = 20 + 4a$

$$\Rightarrow T = 20 + 4\left(\frac{10}{3}\right) = 33.3 \text{ N.}$$

Example 2E2

Masses of 8 kg and 10 kg are connected by a taut, light, inextensible string which passes over a smooth light fixed pulley C as shown in the diagram. The 8 kg mass lies on a rough horizontal plane and the coefficient of friction between the 8 kg mass and the plane is ¼. The 10 kg mass lies on a smooth plane which is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{4}{3}$.

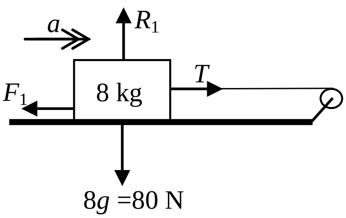


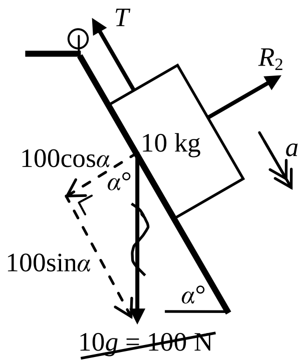
The system is released from rest.

(i) Show on separate diagrams the forces acting on each particle.
(ii) Find the common acceleration of the masses.
(iii) Find the tension in the string.

Solution:

(i)





(ii) Resolving vertically: $R_1 = 80$ N.

$F = \mu R, \quad \Rightarrow F_1 = 0.25(80) = 20$ N.

The equations of motion of the particles, $\vec{F} = m\vec{a}$:

8 kg $\rightarrow$: $T - 20 = 8a$ **A**

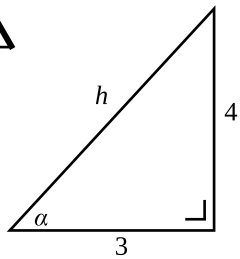
10 kg $\searrow$: $100\sin\alpha - T = 10a$

$\Rightarrow 100\left(\frac{4}{5}\right) - T = 10a$

$\Rightarrow 80 - T = 10a$ **B**

A + **B**: $60 = 18a, \quad \Rightarrow a = \dfrac{60}{18} = \dfrac{10}{3}$ m s^{-2}

(iii) In **A**: $T = 20 + 8a = 20 + 8\left(\dfrac{10}{3}\right) = 46.7$ N.



$\tan\alpha = \frac{4}{3}$

From Pythagoras:

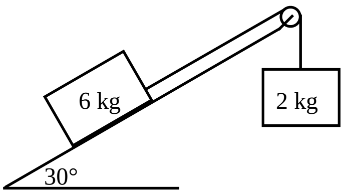
$h^2 = 3^2 + 4^2 = 25$

$\Rightarrow h = 5$

$\Rightarrow \sin\alpha = \frac{4}{5}$

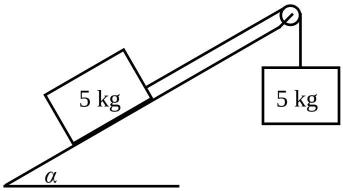
and $\cos\alpha = \frac{3}{5}$

Exercise 2E – force problems with smooth inclined planes

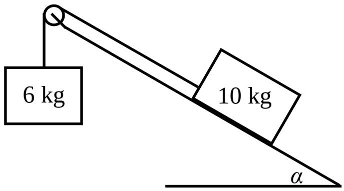


1) Masses of 6 kg and 2 kg are connected by a taut light inelastic string which passes over a smooth light pulley, as shown in the diagram. The 6 kg mass lies on a smooth plane inclined at 30° to the horizontal. The 2 kg mass hangs vertically. The system is released from rest.
   Find: (i) the common acceleration of the particles,
   (ii) the tension in the string.

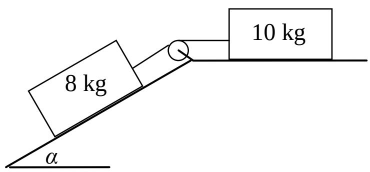
2) Two particles, each of mass 5 kg, are connected by a taut light inelastic string which passes over a smooth light pulley, as shown in the diagram. One 5 kg mass lies on a smooth plane inclined at α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The other 5 kg mass hangs vertically. The system is released from rest.
   Find: (i) the common acceleration of the particles,
   (ii) the tension in the string.



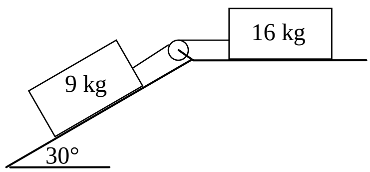
3) Masses of 10 kg and 6 kg are connected by a taut light inelastic string which passes over a smooth light pulley, as shown in the diagram. The 10 kg mass lies on a smooth plane inclined at α to the horizontal, where $\tan\alpha = \frac{4}{3}$. The 6 kg mass hangs vertically. The system is released from rest.
   Find: (i) the common acceleration of the particles,
   (ii) the tension in the string.



4) Masses of 8 kg and 10 kg are connected by a taut, light, inextensible string which passes over a smooth light fixed pulley as shown in the diagram. The 10 kg mass lies on a rough horizontal plane and the coefficient of friction between the 10 kg mass and the plane is 0.4. The 8 kg mass lies on a smooth plane which is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The system is released from rest.
   (i) Show on separate diagrams the forces acting on each particle.
   (ii) Find the common acceleration of the masses.
   (iii) Find the tension in the string.

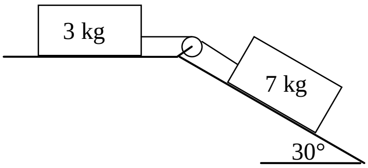


5) Masses of 9 kg and 16 kg are connected by a taut, light, inextensible string which passes over a smooth light fixed pulley as shown in the diagram. The 16 kg mass lies on a rough horizontal plane and the coefficient of friction between the 16 kg mass and the plane is ¼. The 9 kg mass lies on a smooth plane which is inclined at an angle of 30° to the horizontal. The system is released from rest.



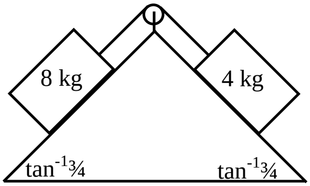
(i) Show on separate diagrams the forces acting on each particle.
(ii) Find the common acceleration of the masses.
(iii) Find the tension in the string.

6) Masses of 3 kg and 7 kg are connected by a taut, light, inextensible string which passes over a smooth light fixed pulley as shown in the diagram. The 3 kg mass lies on a rough horizontal plane and the coefficient of friction between the 3 kg mass and the plane is ⅔. The 7 kg mass lies on a smooth plane which is inclined at an angle of 30° to the horizontal. The system is released from rest.



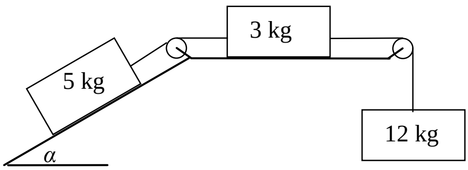
(i) Show on separate diagrams the forces acting on each particle.
(ii) Find the common acceleration of the masses.
(iii) Find the tension in the string.

7) Particles of masses 8 kg and 4 kg are held on the smooth faces of a double inclined plane and connected by means of a light inextensible string passing over a smooth pulley at the top of the plane. The faces of the plane are both inclined at α to the horizontal, where $\tan\alpha = \frac{3}{4}$.



The system is released from rest.
(i) Show on separate diagrams the forces acting on each particle.
(ii) Find the common acceleration of the masses.
(iii) Find the tension in the string.

8) A particle of mass 3 kg, on a smooth horizontal plane, is connected by two light inelastic strings passing over smooth light pulleys at opposite edges of the plane, to two particles of masses 5 kg and 12 kg. The particle of mass 5 kg is on a smooth plane inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The particle of mass 12 kg is hanging freely. The system is released from rest.



(i) Show on separate diagrams the forces acting on each particle.
(ii) Find the common acceleration of the masses.
(iii) Find the tensions in the two strings.

Section 2F – Rough Inclined Planes

This is very similar to the previous section, with the addition that the inclined surfaces here are rough. Therefore, in order to find the friction force on a particle on a rough inclined plane, the normal reaction to the plane must be found. This is found by resolving the forces perpendicular to the plane - the resultant force in the direction perpendicular to the plane must be zero since the motion only takes place in the direction along the inclined plane.

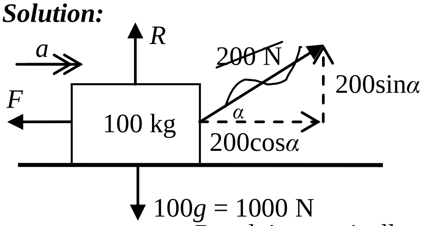
Solution Strategy for questions with objects sliding on rough inclined planes:

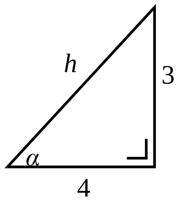
1) **Draw a detailed diagram** of the system, making sure to include all forces, masses and accelerations.
2) **Resolve perpendicular to the plane** for any masses on rough planes to find the normal reactions R. Use $\boldsymbol{F = \mu R}$ to calculate any friction forces.
3) **Work out the Equation of Motion** for each mass in the system. For any particles on an inclined plane, the direction of the equation of motion is along the plane.
4) Solve these equations using **simultaneous equations**.

Example 2F1

A sled of mass 100 kg is pulled by a rope which is inclined at an angle of α above the horizontal, where $\tan\alpha = \frac{3}{4}$, across a horizontal snow covered plateau. The coefficient of friction between the sled and the snow is 0.1. Find the acceleration of the sled if the tension in the rope is 200 N.

Solution:





$\tan\alpha = \frac{3}{4}$

From Pythagoras:

$h^2 = 3^2 + 4^2 = 25$

$\Rightarrow h = 5$

$\Rightarrow \sin\alpha = \frac{3}{5}$

and $\cos\alpha = \frac{4}{5}$

Resolving vertically: $R + 200\sin\alpha = 1000$

$$\Rightarrow R = 1000 - 200\left(\tfrac{3}{5}\right) = 880 \text{ N}$$

$$F = \mu R, \quad \Rightarrow F = 0.1(880) = 88 \text{ N}.$$

The equation of motion of the sled $\rightarrow$, $\vec{F} = m\vec{a}$:

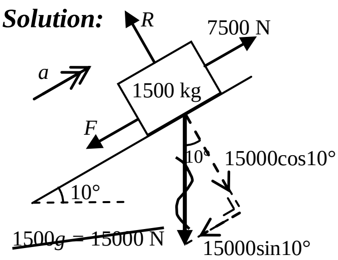
$\Rightarrow 200\cos\alpha - 88 = 100a, \qquad \Rightarrow 200\left(\tfrac{4}{5}\right) - 88 = 100a$

$\Rightarrow 72 = 100a, \qquad \Rightarrow a = 0.72 \text{ m s}^{-2}$

Example 2F2

A car of mass 1500 kg is climbing a hill inclined at 10° to the horizontal. The engine of the car provides a tractive force of 7.5 kN. The coefficient of friction between the car and the hill is ¼. Find the acceleration of the car up the hill, correct to 2 places of decimals.

Solution:



Resolving perpendicular to the plane:

$$R = 15000\cos 10^\circ = 14,772 \text{ N}.$$

$$F = \mu R, \quad \Rightarrow F = 0.25(14,772) = 3,693 \text{ N}.$$

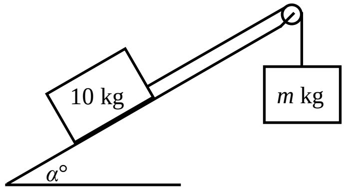
The equation of motion of the car up the plane, $\vec{F} = m\vec{a}$:

$\Rightarrow 7500 - 3693 - 15000\sin 10^\circ = 1500a$

$\Rightarrow 7500 - 3693 - 2605 = 1500a, \qquad \Rightarrow a = 0.80 \text{ m s}^{-2}$

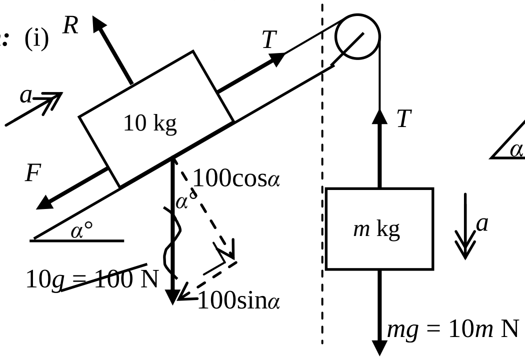
Example 2F3

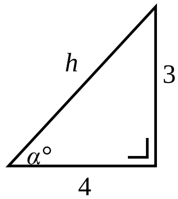
A particle of mass 10 kg is placed on a rough plane inclined at an angle $\alpha°$ to the horizontal, where $\tan\alpha = \frac{3}{4}$. This particle is connected by means of a light inextensible string passing over a. A smooth wedge of mass 3*m* and slope 45° rests on a smooth horizontal surface. A particle of mass *m* is placed o smooth light pulley at the top of the plane to a particle of mass *m* kg, hanging freely under gravity. The coefficient of friction between the 10 kg mass and the plane is ¼. The system is released from rest. The 10 kg mass moves up the plane with acceleration 2 m s^{-2}.



- (i) Show on separate diagrams, the forces acting on the two particles.
- (ii) Find the tension in the string.
- (iii) Show that $m = 12.5$.

Solution: (i)





$\tan\alpha = \frac{3}{4}$

From Pythagoras:

$h^2 = 3^2 + 4^2 = 25$

$\Rightarrow h = 5$

$\Rightarrow \sin\alpha = \frac{3}{5}$

and $\cos\alpha = \frac{4}{5}$

(ii) Resolving perpendicular to the plane:

$R = 100\cos\alpha = 100\left(\frac{4}{5}\right) = 80$ N.

$F = \mu R, \quad \Rightarrow F = 0.25(80) = 20$ N.

The equation of motion of the 10 kg particle up the plane, $\vec{F} = m\vec{a}$:

10 kg ↗: $T - F - 100\sin\alpha = 10a,$

$\Rightarrow T - 20 - 100\left(\frac{3}{5}\right) = 10(2), \qquad \Rightarrow T = 100$ N.

(iii) The equation of motion of the *m* kg particle downwards, $\vec{F} = m\vec{a}$:

m kg ↓: $10m - T = ma, \qquad \Rightarrow 10m - 100 = 2m,$

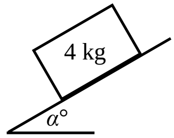
$\Rightarrow 8m = 100, \qquad \Rightarrow m = 12.5$ Q.E.D.

Exercise 2F – force problems with rough inclined planes

1) A horse drags a large log of mass 500 kg across horizontal level ground by means of a rope which is inclined at 30° above the horizontal. The tension in the rope is 3000 N, and the coefficient of friction between the log and the ground is ½.
   - (i) Show all the forces acting on the log in a diagram.
   - (ii) Find the normal reaction between the ground and the log.
   - (iii) Find the acceleration of the log, correct to 2 places of decimals.

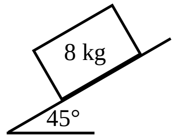
2) A woman pulls a box of mass 80 kg across a horizontal level floor by means of a rope which is inclined at α above the horizontal, where $\tan\alpha = \frac{4}{3}$. The tension in the rope is 500 N, and the coefficient of friction between the log and the ground is ¼.
   (i) Show all the forces acting on the box in a diagram.
   (ii) Find the normal reaction between the floor and the box.
   (iii) Find the acceleration of the box.

3) A particle of mass 4 kg is released from rest and slides down a rough plane which is inclined at an angle $\alpha°$ to the horizontal, where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between the particle and the plane is ¼.



   (i) Show on a diagram all the forces acting on the particle.
   (ii) Find the acceleration of the particle.

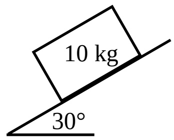
4) A particle of mass 8 kg is placed on a rough plane which is inclined at an angle of 45° to the horizontal. The coefficient of friction between the particle and the plane is μ. The particle is released from rest, and it takes 4 seconds to move a distance of $20\sqrt{2}$ metres down the plane.



   (i) Show on a diagram all the forces acting on the particle.
   (ii) Show that the acceleration of the particle is $\frac{5\sqrt{2}}{2}$ m s^{-2}.
   (iii) Find the value of μ.

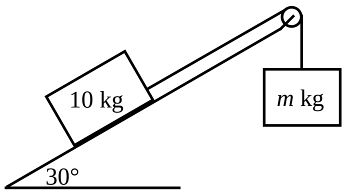
5) A cyclist of total mass 120 kg freewheels down a hill inclined at $\alpha°$ to the horizontal, where $\sin\alpha = \frac{1}{20}$. If the cyclist maintains a constant speed of 12 m s^{-1} on the hill without pedalling, find the resistance (friction) force exerted on the bicycle by the hill.

6) A particle of mass 10 kg is placed on a rough plane inclined at an angle 30° to the horizontal. The particle is on the point of slipping down the plane.



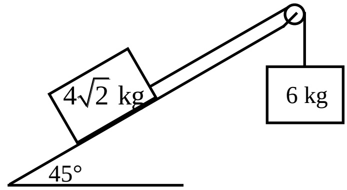
   (i) Show on a diagram all the forces acting on the particle.
   (ii) Find the value of μ, the coefficient of friction between the particle and the plane, giving your answer in the form $\frac{1}{\sqrt{x}}$, $x \in \mathbb{N}$.

A smooth pulley is now attached to the top of the plane. A particle of mass m kg, hanging freely under gravity, is now connected to the particle of mass 10 kg by means of a light inextensible string passing over the smooth pulley at the top of the plane. The particles are released from rest, and the 10 kg particle moves up the plane with an acceleration of 5 m s^{-2}.



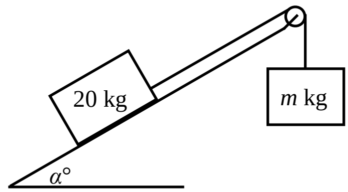
   (iii) Find the value of m and the tension in the string.

7) A particle of mass $4\sqrt{2}$ kg lies on a rough plane inclined at an angle 45° to the horizontal. The coefficient of friction between the particle and the plane is ¼. The $4\sqrt{2}$ kg mass is connected by means of a light inelastic string, which passes over a smooth pulley at the top of the plane, to a particle of mass 6 kg hanging freely under gravity. The system is released from rest, and the 6 kg mass accelerates downwards.



   (i) Show on separate diagrams all the forces acting on each of the particles.
   (ii) Find the common acceleration of the two particles, correct to 2 decimal places.
   (iii) Find the tension in the string, correct to 1 place of decimals.

8) A particle of mass 20 kg is placed on a rough plane inclined at an angle of α° to the horizontal, where $\tan\alpha = \frac{3}{4}$. The particle is connected by means of a light inextensible string passing over a smooth light pulley at the top of the plane to a particle of mass m kg hanging freely under gravity. The coefficient of friction between the 20 kg particle and the plane is ¾. The system is released from rest, and the 20 kg mass accelerates up the plane. The value of the tension in the string is 280 N.



   (i) Find the common acceleration of the particles.
   (ii) Show that m = 35.

Answers to Exercises:

Exercise 2A

1) (i) 0.5 m s^{-2}, (ii) 24,000 N. 2) (i) 0.25 m s^{-2}, (ii) 125 N. 3) (i) 1.5 m s^{-2}, (ii) 75 m.
4) (i) 7 m s^{-2}, (ii) 2 s. 5) (i) 2.5 s, (ii) $\frac{40}{9}$ m s^{-2}, (iii) 0.5 N. 6) (i) 6 m s^{-2}, (ii)5,500 N.
7) 5,600 N.

Exercise 2B

1) (i) $70 - T = 7a,\ T - 30 = 3a$, (ii) 4 m s^{-2}, (iii) 42 N. 2) (i) $T - 10 = a,\ 30 - T = 3a$,
2) (ii) 5 m s^{-2}, (iii) 15 N. 3) (i) $70 - T = 7a,\ T - 50 = 5a$, (ii) $\frac{5}{3}$ m s^{-2}, (iii) 58.3 N.
4) (i) $T - 20 = 2a,\ 50 - T = 5a$, (ii) $\frac{30}{7}$ m s^{-2}, (iii) 28.6 N. 5) (i) 2 m s^{-2}, (ii) $T - 10x = 2x$,
5) (ii) $30 - T = 6$, (iii) 24 N, 2 kg. 6) (i) 2.5 m s^{-2}, (ii) $25 - T = 6.25,\ T - 10x = 2.5x$,
6) (iii) 18.75 N, 1.5 kg. 7) (i) $T - 10x = xa,\ 40 - T = 4a$, (ii) $a = \dfrac{40 - 10x}{x + 4},\ T = \dfrac{80x}{x + 4}$.

Exercise 2C

1) 1.5 m s^{-2}. 2) (ii) $\frac{20}{17}$ m s^{-2}, (iii) 61.76 N. 3) (ii) 2 m s^{-2}, (iii) 16 N.

4) (ii) 4 m s^{-2}, (iii) 48 N. 5) (ii) $\frac{160}{3}$ N, (iii) ⅓. 6) (ii) 45 N, (iii) 5.625 kg.

7) (ii) $S-20=3a,\ T-S-20=6a,\ 60-T=6a$, (iii) $\frac{4}{3}$ m s^{-2}, 24 N, 52 N.

8) (ii) $S-60=8a,\ T-S-5=2a,\ 100-T=10a$, (iii) 1.75 m s^{-2}, 74 N, 82.5 N.

9) (i) 5 m s^{-2}, (ii) ½. 10) (ii) 40 N, (iii) $S-30=3a,\ T-S-40=8a,\ 90-T=9a$,

10) (iv) 1 m s^{-2}, 33 N, 81 N.. 11) 8 m s^{-1}. 12) 80 m.

Exercise 2D

1) $\frac{1}{2}, \frac{\sqrt{3}}{2}, \frac{1}{\sqrt{3}}$. 2) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1$. 3) $\frac{\sqrt{3}}{2}, \frac{1}{2}, \sqrt{3}$. 4) (i) $\frac{3}{5}, \frac{4}{5}$, (ii) $\frac{12}{13}, \frac{5}{13}$, (iii) $\frac{7}{25}, \frac{24}{25}$,

4) (iv) $\frac{15}{17}, \frac{8}{17}$, (v) $\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}$, (vi) $\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}$, (vii) $\frac{1}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$, (viii) $\frac{\sqrt{2}}{\sqrt{3}}, \frac{1}{\sqrt{3}}$, (ix) $\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}$.

5) (i) $\frac{3}{5}, \frac{4}{3}$, (ii) $\frac{12}{13}, \frac{5}{12}$, (iii) $\frac{\sqrt{8}}{3}, \frac{1}{\sqrt{8}}$, (iv) $\frac{\sqrt{45}}{7}, \frac{2}{\sqrt{45}}$, (v) $\frac{\sqrt{2}}{\sqrt{3}}, \frac{1}{\sqrt{2}}$. 6) (i) $\frac{7}{25}, \frac{7}{24}$,

6) (ii) $\frac{\sqrt{3}}{2}, \sqrt{3}$, (iii) $\frac{\sqrt{7}}{4}, \frac{\sqrt{7}}{3}$, (iv) $\frac{15}{17}, \frac{15}{8}$, (v) $\frac{\sqrt{3}}{\sqrt{7}}, \frac{\sqrt{3}}{2}$.

Exercise 2E

1) (i) 1.25 m s^{-2}, (ii) 22.5 N. 2) (i) 2 m s^{-2}, (ii) 40 N. 3) (i) 1.25 m s^{-2}, (ii) 67.5 N.

4) (ii) $\frac{4}{9}$ m s^{-2}, (iii) 44.4 N. 5) (ii) 0.2 m s^{-2}, (iii) 43.2 N. 6) (ii) 1.5 m s^{-2}, (iii) 24.5 N.

7) (ii) 2 m s^{-2}, (iii) 32 N. 8) (ii) 4.5 m s^{-2}, (iii) 52.5 N, 66 N.

Exercise 2F

1) (ii) 3500 N, (iii) 1.70 m s^{-2}. 2) (ii) 400 N, (iii) 2.5 m s^{-2}. 3) (ii) 4 m s^{-2}. 4) (iii) ½.

5) 60 N. 6) (ii) $\frac{1}{\sqrt{3}}$, (iii) 30 kg, 150 N. 7) (ii) 0.86 m s^{-2}, (iii) 54.9 N. 8) (i) 2 m s^{-2}.